PART 4

A Curriculum for All Students

INTRODUCTION

Connected Mathematics® 4 (Phillips et al., 2025) builds on the successes of CMP1 (Lappan et al., 1997), CMP2 (Lappan et al., 2006), and CMP3 (Lappan et al., 2014). CMP4 expands the potential for developing deeper mathematical understanding and thinking for more students. The Connected Mathematics Project (CMP) holds high expectations for its students—all of its students. This belief is reflected in the overarching goal of the curriculum:

> *All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics. This knowledge should include the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency.*

CMP4 is a complete middle grades mathematics curriculum. In the CMP4 Student Edition, the important mathematical concepts and processes are embedded or encoded in the context of the problem. Contextualized problems provide students with opportunities to make sense of the world and empower them to use mathematics to solve problems. The context of the problem helps students develop understanding and skills. It also helps them retrieve and apply their knowledge as needed for future learning. The contexts may be connected to the real world, whimsical, abstract, mathematical, or imaginary. Students and teachers often refer to the problems by their contextual names. The contexts provide opportunities for students to elicit genuine interest and wonderment in the context itself, access to the mathematics problem, anchor instruction in a context to develop understanding, and highlight applications of mathematics in everyday matters. The problems also provide students with opportunities to affirm, value, and build on their experiences through their families' and communities' cultural and linguistic funds of knowledge. This mathematical and pedagogical grounding is valuable for all students.

CMP has produced grade 8 curriculum materials that serves both grade 8 and the first year high school mathematics course with a focus on algebra. This is because of CMP's unique approach to combining conceptual and procedural knowledge and its unique development of algebra and functions into one strand. In an era where educational equity is paramount, our approach to these courses provides students with the same curriculum materials in grade 8 and the first year in high school mathematics. This contrasts other approaches where students use different curriculum materials. Further, students have similar opportunities to talk about the same mathematics yet can advance in their understandings in potentially different ways that are meaningful to their future pathways in mathematics.

The CMP4 Teacher Edition offers support for every problem in the unit on when and how to implement effective strategies for CMP classrooms. CMP classrooms represent a wide range of diversity in terms of gender, race, ethnicity, economic status, or disability. Evidence suggests that CMP can be and has been successfully implemented in classrooms that include students with special needs, academically gifted students, historically and culturally underrepresented students, and multilingual students. We believe that CMP provides all students with opportunities to engage in collaborative learning, to assume leadership roles, and to enhance

self-esteem and self-acceptance. This section shows how effective strategies for diverse populations of students are already built into the CMP curriculum. Particular features of CMP4 such as the Unit Alignment Chart, the At a Glance, the Extended Launch—Explore—Summarize, the Emerging Mathematical Ideas, the Arc of Learning™ Framework, and the Now What Do You Know? are discussed in terms of how they support teachers during implementation of CMP.

For more information, see Part 6, CMP Research, Evaluation, and Awards and the CMP website: https://www.connectedmath.msu.edu/.

Did You Know?

In terms of the evidence of effectiveness of teaching strategies for all students in CMP classrooms, there is a rich database of research and evaluation studies on CMP. These studies are conducted in CMP classrooms that represent diverse classroom settings. The database includes over 550 studies, including research articles, books, book chapters, dissertations, reports, conference proceedings, and essays studies on CMP. Many of these studies involve student learning, equity, diversity, and inclusion studies in mathematics classrooms. The database is interactive so one can search for a particular topic. To access the database, please visit the CMP website at Michigan State University: https://www.connectedmath.msu.edu/.

ATTENDING TO INDIVIDUAL LEARNING NEEDS FRAMEWORK

The *Attending to Individual Learning Needs Framework* (AILN Framework) is a framework for supporting diverse learning needs in rich mathematics problem-solving environments (Edson, Slanger-Grant, & Dole, 2023; Slanger-Grant, Edson, & Phillips, 2020a, 2020b; Slanger-Grant, Edson, & Phillips, 2021; Slanger-Grant, 2020; Slanger-Grant & Dole, 2020; Slanger-Grant & Luczak, 2019). The differentiation framework highlights dimensions needed to enhance how students can access and engage with deep mathematical learning. The framework supports teachers in developing students as knowers and doers of mathematics by focusing on students' strengths, backgrounds, and experiences. As learners need different supports at different times in their learning, teachers attend to these diverse learning needs throughout classroom instruction. The framework contains five dimensions for designing a classroom environment in which teachers can support mathematics learning for each and every student. The five dimensions of the framework include (1) Problem-Solving Environment; (2) Agency, Identity, and Ownership; (3) Portrayal; (4) Language; and (5) Time. In the sections that follow, we expand on each dimension in more detail. In this section, we also expand on specific populations of students, including students with special needs, academically gifted students, historically and culturally underrepresented students, and multilingual students. Note that many of the strategies described for specific populations also work for all students.

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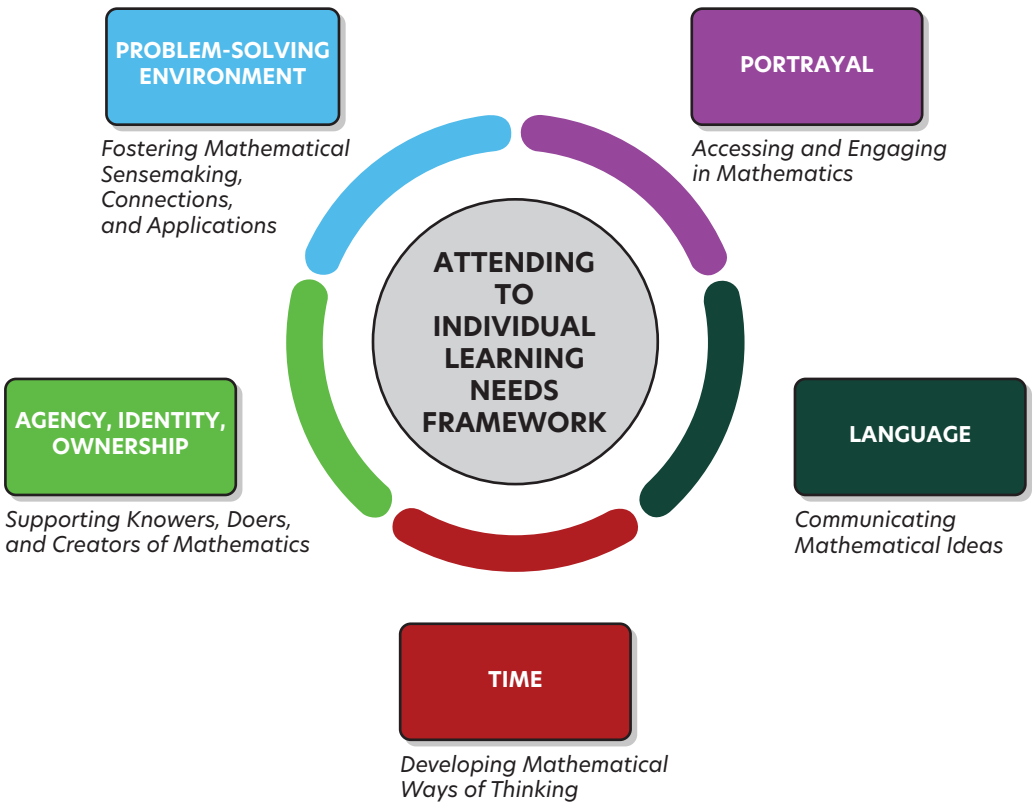
CMP Attending to Individual Learning Needs Framework



The development and use of the Attending to Individual Learning Needs Framework builds on information gained over the 40 years of field testing and on the evidence from the research on the needs of learners. This includes research in CMP classrooms that focuses on students with special needs, academically gifted students, historically and culturally underrepresented students, and multilingual students. It also includes research and policy efforts on equity, diversity, and inclusion. The Attending to Individual Learning Needs Framework provides a single teacher resource that leverages what is known in mathematics education in these different yet important areas needed for the teaching and learning of mathematics.

We envision that the Attending to Individual Learning Needs Framework is useful for teachers in two distinct ways:

- It links what is known about research in mathematics classrooms to classroom practice.
- It communicates how teachers meet individual learning needs in CMP classrooms.

First, the Attending to Individual Learning Needs Framework highlights key research that supports many teacher actions as contributions to attending to the needs of learners. The research we used focuses on ways to work around student challenges. The idea is to enhance and broaden opportunities for students to make sense of mathematical ideas.

Second, the Attending to Individual Learning Needs Framework is a way to make explicit how teachers meet the needs of learners. Teachers use their own strategies and routines and the ones suggested in the Teacher Edition to meet the needs of learners. Many of these teacher moves are directly related to the individual needs of students. Teachers need to know when and be confident in communicating that they are strategically meeting the needs of each and every learner.

Each problem in CMP4 Teacher Edition gives guidance pertaining to the components of the Attending to Individual Learning Needs Framework and pedagogical strategies that support the diverse needs of learners.

Problem-Solving Environment

PROBLEM-SOLVING ENVIRONMENT	
Fostering Mathematical Sensemaking, Connections, and Applications	*Opportunities for students to solve problems and **make sense of the embedded mathematics and to connect and apply these understandings** to develop deeper mathematical understandings and ways of thinking* Characteristics: • allows multiple entry points • provides supports that assists them in the problem. • includes ways to connect experiences and thinking to the problem • provides supports to communicate reasoning with others • fosters physical and social space to explore and solve problems • allows welcoming, affirming feeling while having high expectations • provides resources and time for mathematical ideas and concepts

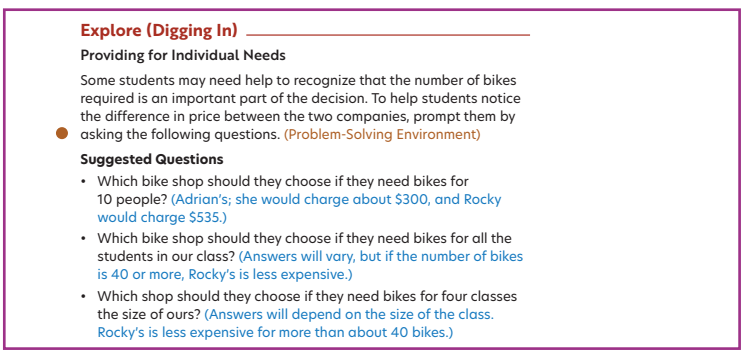
A problem-solving environment allows multiple entry points with opportunities for students to develop deep mathematical understandings. In this environment, students may benefit from supports that assist them in understanding elements of a task, ways to connect their experiences and thinking to the questions asked, or how to communicate their reasoning to others. In particular, the new CMP STEM problem-based format, which includes three parts, Initial Challenge, What If . . . ?, and Now What do You Know?, provides the elements to create an environment needed to develop students' mathematical reasoning and understanding.

The problem-solving environment allows the physical and social space for students to explore and solve mathematics problems. It is an environment that is welcoming and affirming while fostering high expectations and focused instruction. Teachers can use resources and time to support students in exploring mathematical ideas and concepts and to share their ideas with others. Table 4.1 provides questions that can support students' engagement in a problem-solving environment.

Table 4.1. Teacher Questions for the Problem-Solving Environment

Questions to Help Students Access the Problem-Solving Task	Suggestions for Students to Access the Problem
1. What do you know about the question? OR What is the question?	• sort out what needs to be done
2. What do you know?	• search for resources in what is given and organize information
3. Do you have any guesses/conjectures/estimates for the answer?	• test ideas toward a solution path
4. What information do you need to find the answer?	• sort out any questions they may have
5. Is this like anything that you have done before?	• connect to and/or see relationships to prior mathematical learning
6. If you knew what to do, what would you do?	• take risks toward or begin a partial solution path
7. Have you thought about . . .? How might this help?	• develop flexible ways of thinking

Example from Grade 6 Variables and Patterns Unit: Problem 2.1. In this problem, students are comparing the cost of renting bicycles from two different companies. The rental cost information for one company is provided in a table, and the information for the other company is provided in a graph. The Initial Challenge poses an open-ended question about which company to choose. To support students in making sense of the mathematics embedded in this challenge, they may benefit from the Suggested Questions provided in this example from the Teacher Edition.



Explore (Digging In)

Providing for Individual Needs

Some students may need help to recognize that the number of bikes required is an important part of the decision. To help students notice the difference in price between the two companies, prompt them by asking the following questions. (Problem-Solving Environment)

Suggested Questions

- Which bike shop should they choose if they need bikes for 10 people? (Adrian's; she would charge about $300, and Rocky would charge $535.)
- Which bike shop should they choose if they need bikes for all the students in our class? (Answers will vary, but if the number of bikes is 40 or more, Rocky's is less expensive.)
- Which shop should they choose if they need bikes for four classes the size of ours? (Answers will depend on the size of the class. Rocky's is less expensive for more than about 40 bikes.)

Agency, Identity, and Ownership

AGENCY, IDENTITY, AND OWNERSHIP	
Supporting Knowers, Doers, and Creators of Mathematics	*Opportunities for students to **participate as knowers, doers, and creators of mathematics** to develop deep mathematical understandings* Characteristics: • showcases student work and voices to highlight mathematical thinking • encourages reflection on individual and collective thought processes • utilizes student work from prior classes as examples of mathematical thinking • promotes personal connections to mathematical contexts • integrates current events, cultural elements, and local figures into discussions • actively invites student contributions during class discussions

Since the problems in CMP are contextual, they appeal to students' curiosity about the world they live in. The problems provide the opportunity to

- make the mathematical ideas accessible and relatable to the real world;
- validate students' knowledge and multifaceted contributions to learning;
- use emerging mathematical thinking to develop more complete conceptual knowledge; and
- promote curiosity and exploration of ideas.

Teachers can adjust the problem contexts for personalization and provide student work to generate discussion and advance the opportunities for learning mathematical ideas. Teachers can positively position their students as mathematicians by using student work and student voice to highlight mathematical thinking and encourage students to reflect on their thought processes and those of others. In addition to the student work produced in class, there are many examples of students thinking embedded in the problems. These examples come from students in prior CMP classes.

Encouraging a personal connection to the mathematical contexts can support students engaging in the mathematical ideas. Teachers can include current events, cultural elements, and local figures in the classroom and invite students to contribute their ideas during class.

Example Grade 6 Comparing Quantities Unit: Problem 2.1. This problem uses the packaging of cupcake context as a way for students to use reasoning with ratios, also called proportional reasoning, to solve problems. This will help students develop additional strategies and experience with finding equivalent ratios. The primary goal is to showcase student work and thinking for students in your classroom. The What If . . . ? situations provide opportunities to analyze student strategies for scaling up ratios and new cupcake packaging ideas if these strategies haven't already been suggested by your own students.

> As you listen to groups share their strategies, take note of interesting strategies to share in the Summarize. Students may use reasoning like the characters (Zane and Bruce) in What If . . . ? Situation A. If so, you can highlight your students' work instead of focusing on Zane and Bruce. (Agency, Identity, Ownership)

Portrayal

PORTRAYAL	
Accessing and Engaging in Mathematics	*Opportunities for students to **engage, represent, and interpret with mathematics in a variety of ways** to develop deep mathematical understandings* Characteristics: • offers multiple access points to problems to enhance student connection and engagement with mathematical ideas • allows students to develop and represent their understanding in various formats • utilizes images as anchors for embedding and retrieving understanding of key mathematical concepts • presents diverse examples to support the portrayal of mathematics

Providing students with multiple access points to the problems increases the likelihood for students to connect and engage with the mathematical ideas. The curriculum also provides the opportunity to develop and represent their understanding in a variety of ways. For example, graphic organizers can be used by the teacher to present information or by the students to organize information and to compare and contrast concepts and ideas. Graphic organizers such as word clusters, rebuses, and key terms charts can be used to support development of language. Venn diagrams, concept maps, and other techniques can help students organize information. You will find graphic organizers titled as Learning Aids and Teaching Aids in the Teacher Edition. Learning Aids and Teaching Aids are tools that students and teachers can use during the problem. Some Learning Aids are time savers, repeating something as it is in the corresponding Student Edition so that students spend less time setting up the presentation of their strategies and more time thinking through their strategies. Teaching Aids include information showing different strategies and examples or elaborations of ideas. Often the information embedded in the Learning and Teaching Aids are anchors for embedding and retrieving understanding of important ideas. Table 4.2 provides different ways to portray the mathematics in classrooms.

Table 4.2. Portrayals of Mathematics in CMP Classrooms

Portrayals of the Mathematics		
Visual	**Audio**	**Kinesthetic**
• Use diagrams, pictures, posters, and anchor charts. • Write or diagram what is being narrated or gestured. • Use of color or spacing to bring focus to key words and ideas. • Connect multiple representations (tables, graphs, or symbols) of an idea. • Access technology.	• Use multiple explanations of a single concept. • Narrate what is being displayed or gestured. • Provide informal explanations. • Give students opportunities to listen to each other explain their mathematical thinking. • Pause to allow students to process their thinking. • Access technological devices to support narratives or audio.	• Use manipulatives. • Have students work with multiple partners in small-group work. • Reduce the number of distractions in the classroom environment. • Use movement and gestures to help express what is displayed or narrated, such as using your arms to "make axes in the air" or showing the shape of a curve with your hands. • Have students imagine a contextual scene or experience.

Example from Grade 7 Comparing and Scaling Unit: Problem 1.2. This problem gives students the opportunity to practice the scaling strategies they explored earlier. The Initial Challenge goes back to the orange juice recipes from Problem 1.1. Students choose part-to-part or part-to-whole ratios, as appropriate. In the What If . . . ? situations, students will look at scaling strategies to solve proportions, foreshadowing the next problem, where proportions are defined and solving proportions is dealt with directly. This problem encourages students to represent their thinking using various representations.

Suggested Questions

- How are the strategies used in our classroom similar to and different from each other? (Answers will vary.)

Students may use rate tables (like Aaliyah in the What If . . . ?), equations to scale (like Zaire in the What If . . . ?), or other unique strategies. (Portrayal) One possibility includes drawing pictures to show all of the relationships in the recipe.

Language

LANGUAGE	
Communicating Mathematical Ideas	*Opportunities for students to **communicate mathematical ideas** to develop deep mathematical understandings* Characteristics: • provides multiple opportunities for students to express and share mathematical thinking • acknowledges the social nature of math learning and problem solving • encourages students to articulate their mathematical thoughts • utilizes diverse forms of communication to deepen understanding • enhances mathematical fluency through active engagement in problem solving and discussion

Providing students with multiple opportunities to produce and communicate representations of their mathematical thinking increases their chances to engage with the material.

Learning and doing mathematics suggest a social nature while exploring, generating, justifying, representing, and solving problems. Learners need ways to communicate their thinking to others. Being immersed in communicating about mathematics in various ways opens access to developing an understanding of deep mathematical relationships. Learners strengthen their mathematical fluency by doing, writing, and speaking about mathematics. Often, learning of new concepts involves different portrayals that can then serve as the visual representation of the concept.

Specific examples of developing mathematical language include

- linking mathematical notation to words, context, or other symbols;
- making sense of new contextual situations;
- attaching mathematical key terms to descriptions of ideas;
- relating one mathematical object to another; and
- activating prior learning or relating to background knowledge.

Teachers can model and encourage how to communicate mathematical ideas through discussion, drawing diagrams, and writing. Teachers can prompt written and verbal communication with question starters, sentence starters, and diagram starters. For example, teachers can use the following five guidelines for simplifying language: (1) use short sentences and eliminate extraneous information, (2) change pronouns to nouns, (3) underline key points or key terms, (4) turn narratives to lists, and (5) use charts or diagrams. Teachers and students can generate displays that link mathematical language and concepts to examples, such as anchor charts or word walls.

Example from Grade 8 Thinking with Mathematical Models Unit: Problem 2.3. This problem introduces the term *slope* and gives students opportunities to practice developing linear equations from information given by graphs and tables of values. In the Initial Challenge, students will look at graphs, tables, and descriptions and by finding the slope and *y*-intercept write linear equations. In the What If . . . ? situations, students will analyze student claims to further understanding of the idea of slope. This problem provides students with the opportunity to link the key term *slope* to various representations and to express and share their reasoning with others.

Presenting the Challenge

Point out that it is usually easy to figure out the *y*-intercept of a graph and thus the number *b* in an equation with form $y = mx + b$, but so far students have only rough ideas of what *m* tells about the graph (something about its direction up or down and its steepness). The goal of this problem is to learn how to use a mathematical measure of steepness called **slope**. (Language)

Time

TIME	
Developing Mathematical Ways of Thinking	*Opportunities for students to **develop mathematical ways of thinking** to develop deep mathematical understandings* Characteristics: • allows student exploration, sharing, and summarization in problem solving • embeds design elements for understanding mathematical concepts • provides a sequence of problems centered on key mathematical concepts • aids in developing conceptual and procedural knowledge, moving from informal to formal understanding over problems, units, and grades

In addition to allowing time for students to explore, share, and summarize their thinking as they solve a problem, other design features of CMP allow embedded opportunities for students to develop sensemaking of mathematical ideas through a sequenced set of problems that build toward a big idea. Students can develop conceptual and procedural knowledge from informal to more formal development over time.

Over time, students need to develop from informal to formal.

- The connections within and across the units of CMP support developing deep mathematical understandings that grow and build over time.
- The Arc of Learning™ guides teachers' strategies for developing knowledge of a big idea(s) over a unit.
- The Now What Do You Know? support students' summarizing and organizing daily activities into essential takeaways.
- The Mathematical Reflections support students' metacognition and reflections on a collection of classroom experiences in order to connect their mathematical learning to other experiences.

Example from Grade 7 Completely Rational Unit: Problem 1.3. In this problem, students model addition and subtraction on chip board displays. Students explore relationships between addition and subtraction as well. One chip color indicates positive values, and the other chip color indicates negative values. This problem promotes informal experiences with addition and subtraction of signed numbers that, over time during this unit, will be formalized as algorithms that students deeply understand.

> **Implementation Note:** This problem may take 2 days for students to fully explore the ideas and to discuss their findings as a whole class. Giving them this time to investigate creating zeroes will be beneficial when they use it to create algorithms in Investigation 2. (Time)

Each problem in CMP4 Teacher Edition gives guidance pertaining to the components of the Attending to Individual Learning Needs Framework and pedagogical strategies that support the diverse needs of learners. The unit alignment chart contains the mathematical goals, the Arc of Learning™ phase, Now What Do You Know?, and the Emerging Mathematical Understandings. For each problem, this chart indicates where it fits within the Arc of Learning™ Framework. The Arc of Learning™ Framework in the provides the teachers a guide as to how the mathematical understandings evolve from informal to more sophisticated understandings. The chart also highlights the Now What Do You Know? in the Student Edition that connects the embedded understandings with prior and future knowledge. It provides student-facing questions for students to self-assess and consolidate their learning. The Emerging Mathematical Ideas listed in the chart bring all of the ideas together by listing the mathematics that students will be using as they answer the Now What Do You Know? The At a Glance provided for each problem and Mathematical Reflection includes teacher moves, key pieces from the Extended Launch–Explore–Summarize, and organizational support. The Extended Launch, Explore, and Summarize provides teachers with questions to ask students at each stage of the problem to support student learning and support ongoing formative assessment. Actual classroom scenarios and examples of student thinking are included in the Launch, Explore, and Summarize sections to help stimulate teachers' imaginations about what is possible. In the Teacher Edition, teachers are provided with the answers alongside each problem and mathematical reflection from the Student Edition. The answers discuss how students might think about the problem and include examples of student thinking and strategies, and support teachers in how students access problems, employ different strategies, and reach solutions.

CMP AND STUDENTS WITH SPECIAL NEEDS

The Attending to Individual Learning Needs Framework helps to provide a curriculum appropriate for students with special needs. We recognize the tremendous diversity that characterizes students with disabilities, including emotional disabilities (e.g., mental illness, anxiety and depression, behavior disabilities), physical disabilities (e.g., blindness, deafness, cerebral palsy, mobility impairments), intellectual disabilities, autism, speech and language impairments, attention deficit hyperactivity disorder, and specific learning disabilities (e.g., reading dyslexia, writing dysgraphia, math dyscalculia). While many students with special needs are eligible to receive special education services provided by public schools, we also recognize the challenges of speaking about students with special needs as a group in a meaningful way.

For each problem in CMP4, there are suggestions in the Teacher Edition for students with special needs that relate to the components of the Attending to Individual Learning Needs Framework. There are also pedagogical strategies to support teachers when making instructional decisions for students with special needs. The unit alignment chart contains the mathematical goals, the Arc of Learning™ phase, Now What Do You Know?, and the Emerging Mathematical Ideas. For each problem, this chart indicates where it fits within the Arc of Learning Framework. The Arc of Learning™ Framework provides the teachers a guide as to how the mathematical understandings evolve from informal to more sophisticated understandings. The chart also highlights the Now What Do You Know? in the Student Edition that connects the embedded understandings with prior and future knowledge. It provides student-facing questions for students to self-assess and consolidate their learning. The Emerging Mathematical Ideas listed in the chart bring all of the ideas together by listing the mathematics that students will be using as they answer the Now What Do You Know? In the Teacher Edition, teachers are provided with the answers alongside each problem and mathematical reflection from the Student Edition. The answers discuss how students might think about the problem and include examples of student thinking and strategies, and support teachers in how students access problems, employ different strategies, and reach solutions.

> **“There are also pedagogical strategies to support teachers when making instructional decisions for students with special needs.”**

The conceptual framework upon which CMP is built involves sound teaching principles and practices for students, which is essentially the same foundation for working with special populations. CMP was developed at the beginning with the belief that calculators should be made available to students, which aligns with accommodations that many special-needs students are given. Furthermore, CMP incorporates manipulatives in its curriculum. While it is stressed within the CMP framework that manipulatives are to be used only when they can help students develop an understanding of mathematical ideas, it should be clear that students with special needs may need to use manipulatives more often than their general education peers. Some may claim CMP poses a challenge for special-needs students due to its language-based problem situations; however, the CMP curriculum incorporates many recommendations by researchers in the field of special education. Embedded strategies and collaborative learning groups assist in making mathematics accessible to special-needs students.

CMP uses real-life problems, a pedagogical technique repeatedly stressed in reaching special-needs students in mathematics classrooms. Guiding the development of CMP was an emphasis on making meaningful connections for students, among various mathematical topics and between mathematics and other disciplines. Maccini and Gagnon (2000) demonstrated that

embedding problems within real-world contexts improves the motivation, participation, and generalization for special-needs students. Other practices within the CMP framework that facilitate teaching mathematics to all students, including those with special needs, include repetition and review, keeping expectations high, and teaching conceptual knowledge. The ACE section at the end of every investigation allows students to tackle additional exercises from the unit as well as to work on problems connected to earlier units.

CMP provides opportunities for students to work in small groups and pairs, as well as with the whole class or individually. Educational research suggests that collaborative groups can be beneficial to all students; however, some attention should be paid to the groupings to ensure that students with special needs are able to participate actively. While studies have shown that collaborative learning has positive benefits on students' motivation, self-esteem, cognitive development, and academic achievement, the very dynamic of these learning methods may exclude special education students due to their disparities in skills, such skills as content area, communication, and social abilities (Brinton, Fujiki, & Montague, 2000). In discussing the structure of collaborative groups, researchers stress the importance of providing opportunities for all students, including students with special needs (or any diverse learners) to participate.

For more teacher strategies for implementing the CMP4 curriculum, please see Guide Aid 2.E, Pedagogical Strategies in Part 2, Building on the Successes of CMP1, CMP2, and CMP3—What's in CMP4?

CMP AND ACADEMICALLY GIFTED STUDENTS

The Attending to Individual Learning Needs Framework helps to provide a curriculum appropriate for academically gifted students. Particular features of CMP4 support academically gifted students. For each problem in the CMP4 Teacher Edition, teachers are provided support for academically gifted students. For each problem and mathematical reflection, the At a Glance includes teacher moves, key pieces from the Extended Launch—Explore—Summarize, and organizational support for supports. The Extended Launch - Explore - Summarize includes questions to ask students at each problem phase are included to help teachers support student learning and support ongoing formative assessment. Actual classroom scenarios and examples of student thinking are included in the Launch, Explore, and Summarize sections to help stimulate teachers' imaginations about what is possible. In the At a Glance and Extended Launch Explore Summary, there is support for teachers when making instructional decisions in ways that connect explicitly to the Attending to Individual Learning Needs Framework and the Pedagogical Strategies. There are also questions in the Launch—Explore—Summarize sequence labeled Going Further that teachers can pose to students who are ready to advance. In the ACE assignments, the Extensions often go beyond what was done in the classroom; they can be used as additional exercises to push students' thinking. Along with the deep real-world mathematical situations offered in this curriculum, these features provide all students, including gifted students, with opportunities to explore challenging real-life problems in their daily lives. The unit alignment chart contains the mathematical goals, the Arc of Learning™ phase, Now What Do You Know?, and the Emerging Mathematical Ideas. For each problem, this chart indicates where it fits within the Arc of Learning™ Framework. The Arc of Learning™ Framework provides the teachers a guide as to how the mathematical understandings evolve from informal to more sophisticated understandings. The chart also highlights the Now What Do You Know? in the Student Edition that connects the embedded understandings with prior and future knowledge. It provides student-facing questions for students to self-assess and consolidate their learning. The Emerging Mathematical Ideas listed in the chart bring all of the ideas together by listing the mathematics that students will be using as they answer the Now What Do You Know? In the Teacher Edition,

teachers are provided with the answers alongside each problem and mathematical reflection from the Student Edition. The answers discuss how students might think about the problem and include examples of student thinking and strategies, and support teachers in how students access problems, employ different strategies, and reach solutions.

Adjustments in both the material and learning environment may be necessary. Maker and Nielson (1995) describe such modifications in content and process. Content modifications for the CMP classrooms include the following:

- Students need a variety of problems to work on.
- The content of the curriculum needs to be organized around key concepts or abstract ideas, rather than some other organization (as noted by Bruner, 1977).
- Problems should be complex, and students should be pushed to abstraction. (Additional opportunities to support abstraction are described in the teacher support, particularly in the Going Further features described below.)

Process modifications for the CMP classrooms include the following:

- Promote higher levels of thinking by stressing use rather than acquisition of information. (Students continue to use information from previous units in the current unit they are studying.)
- Provide open-ended questions in order to stimulate divergent thinking and to "contribute to the development of an interaction pattern in which learning, not the teacher, is the focus" (Maker & Nielson, 1995).
- Facilitate student inquiry of content and encourage questions. (Problems often ask students to think about the questions of why and how.)
- Offer opportunities for students to express their reasoning. (Students are constantly asked to explain or justify their responses in CMP.)
- Make group interaction a regular part of the curriculum for gifted students to enable them to develop social and leadership skills.

CMP is designed so that many of the modifications described by Maker and Nielson are embedded in the curriculum. Other simple modifications are possible in order to support gifted students and still maintain the integrity of the curriculum. For example, Reis and Renzoulli (2003) discuss the Schoolwide Enrichment Model (SEM), which can be used to promote challenging and high-end learning in schools. A SEM modification accommodates the needs of the gifted student and offers suggestions on how to adjust the level, depth, and enrichment opportunities provided by a curriculum.

CMP offers students rich experiences with a variety of mathematical content. Students are introduced to important areas of mathematics, such as probability, statistics, and transformational and Euclidean geometry, early in their career so that they can see the vast terrain of mathematics. The algebra strand in CMP is organized around functions, which are the cornerstone of calculus, and the structure of the real numbers, which brings coherence to the exploration of algebraic ideas. The emphasis in grade 8 is on algebra and functions. As described earlier, the units are organized so that there are two pathways using the same curriculum materials to achieve either grade 8 mathematics or the first-year high school courses focused on algebra.

For more teacher strategies for implementing the CMP4 curriculum, please see Guide Aid 2.E, Pedagogical Strategies in Part 2, Building on the Successes of CMP1, CMP2, and CMP3—What's in CMP4?

CMP AND HISTORICALLY AND CULTURALLY UNDERREPRESENTED STUDENTS

The Attending to Individual Learning Needs Framework acknowledges and helps to address issues of equity, diversity, and inclusion in mathematics classrooms. It acknowledges and helps disrupt the historical and contemporary patterns of inequities that negatively impact historically and culturally underrepresented students. To improve students' learning experiences, broadening access and participation to mathematics for all students and teachers in mathematics classrooms is critical. The Attending to Individual Learning Needs Framework helps to foster a welcoming and affirming environment that has high expectations and supports rigorous mathematics instruction.

For each problem in CMP4, there are suggestions in the Teacher Edition for historically and culturally underrepresented students that relate to the components of the Attending to Individual Learning Needs Framework. There are also pedagogical strategies to support teachers when making instructional decisions for historically and culturally underrepresented students. The unit alignment chart contains the mathematical goals, the Arc of Learning™ phase, Now What Do You Know?, and the Emerging Mathematical Ideas. For each problem, this chart indicates where it fits within the Arc of Learning™ Framework. The Arc of Learning™ Framework provides the teachers a guide as to how the mathematical understandings evolve from informal to more sophisticated understandings. The chart also highlights the Now What Do You Know? in the Student Edition that connects the embedded understandings with prior and future knowledge. It provides student-facing questions for students to self-assess and consolidate their learning. The Emerging Mathematical Ideas listed in the chart bring all of the ideas together by listing the mathematics that students will be using as they answer the Now What Do You Know? In the Teacher Edition, teachers are provided with the answers alongside each problem and mathematical reflection from the Student Edition. The answers discuss how students might think about the problem and include examples of student thinking and strategies, and support teachers in how students access problems, employ different strategies, and reach solutions. CMP is a complete and inclusive curriculum and assessment program that employs an instructional model to promote a collaborative, inquiry-oriented, student-centered environment. There are regular opportunities for formative assessment. The emphasis on inquiry-based learning can support classroom environments to be more equitable (Tang, et al., 2017). CMP supports equity, diversity, and inclusion in its design in several ways.

- **CMP tasks are contextualized, providing students with opportunities to make sense of the world and empowering them to use mathematics to solve problems.** In CMP, the mathematics that students make sense of are embedded within relevant and authentic contextualized situations. Contextualized problems provide opportunities for students to elicit genuine interest and wonderment in the context itself, access to the mathematics problem, anchor instruction in a context to develop understanding, and highlight applications of mathematics to everyday matters and generating interest in mathematics. For example, middle school problems in CMP focus on fairness and help to provide students with opportunities to study the inequities involved in different situations. These problems also provide students with opportunities to affirm, value, and build on their experiences through their families' and communities' cultural and linguistic funds of knowledge (González et al., 2006). In CMP, if time is spent exploring interesting mathematics situations, reflecting on solution methods, examining why the methods work, and comparing methods to previous situations, then students are likely to build and use more robust understanding of mathematical concepts and related procedures to solve relevant and authentic problems.

- **Mathematics learning is focused on big ideas of mathematics developed through connected sequences of problems, mathematical reflections, and Applications—Connections—Extensions homework.** Students' perceptions about a discipline come from the problems with which they are asked to engage. In CMP, important mathematical ideas are embedded in the context of interesting problems. In CMP, students spend significant portions of their mathematics time solving problems that require thinking, planning, reasoning, computing, and evaluating. As students explore a series of connected problems, they develop an understanding of the embedded ideas and, with the aid of the teacher, abstract powerful mathematical ideas, problem-solving strategies, and ways of thinking. Through problem solving, students make meanings of mathematics in their own ways drawing on their prior understanding and cultural experiences (Turner & Drake, 2016).
- **The problems are open and provide students with opportunities to engage in important mathematics, reasoning, sensemaking, and justification.** Each CMP problem has some or all of the following characteristics: (a) embeds important, useful mathematics; (b) promotes conceptual and procedural knowledge; (c) builds on and connects to other important mathematical ideas; (d) requires higher-level thinking, reasoning, and problem solving; (e) provides multiple access points for students; (f) allows for various solution strategies; and (g) creates an opportunity for the teacher to assess student learning. This provides "low floor, high ceiling" opportunities for students to access mathematics at different levels (Boaler, 2016), use different representations to show their thinking, and communicate their understandings in different ways.
- **The problems invite student questions, conversations, and conjectures about mathematics.** Problem-based teaching and learning opens the mathematics classrooms to exploring, conjecturing, reasoning, and communicating. Explaining their own strategies and examining others' thinking help students and teachers learn from each other about different ways of thinking (Bieda & Staples, 2020). During the Launch of the problem, students make sense of the context and the challenge presented by the problem. In the Explore phase of the problem, students observe patterns and relationships; recognize and employ prior understandings to test, discuss, verbalize, and generalize patterns; justify their reasoning, including the expectation to make sense of their own thinking, the thinking of others, and the thinking of the discipline; and make judgements on what operation or representation is useful in a situation as well as carrying out any needed computation(s). During the Summarize phase of the problem, students clarify their strategies and critique the reasoning of others; make visible the mathematics embedded in the problem; represent and communicate ideas; and make sense of and connect mathematics from their experiences to prior learnings. In CMP classrooms, students discuss mathematical reasoning with themselves, each other, the teacher, and the whole class.
- **CMP has a welcoming and affirming environment.** The CMP curriculum offers engaging problems in real-world contexts. Each problem begins with an idea or task that provides multiple entry points for students with various strengths.
- **CMP has high expectations and rigorous instruction.** The problem-based curriculum provides contexts that allow for students to solve problems in multiple ways. CMP classrooms depend on a respectful discourse necessary to students becoming more critical thinkers and collaborative problem solvers. CMP promotes students as doers, collaborators, knowers, creators, and communicators of mathematics.
- **CMP curriculum and assessment are inclusive.** CMP employs a Launch—Explore—Summarize pedagogical model to promote a collaborative inquiry environment. Opportunities for regular formative assessment exist throughout all units.

- **CMP provides ongoing professional learning.** CMP has offered and provided professional learning opportunities for teachers for more than 40 years during the summer and academic year, including workshops, conferences, video conferencing chats, webinars, and various social media outlets. CMP leaders have engaged in professional development with teachers across the United States as well as around the world. CMP also hosts an extensive website that continuously supports teachers and teacher leaders.

Note that this is also found in Guide Aid 2.D, CMP4 and Culturally Responsive Teaching in Part 2, Building on the Successes of CMP1, CMP2, and CMP3—What's in CMP4?

It is also necessary to consider how students' cultural backgrounds and previous experiences might affect their ability or willingness to participate in an assessment activity. "Because schooling practices tend to conform more or less to middle-class European-American experiences and values, students from other cultural backgrounds may be misassessed by virtue of cultural and other experiential differences" (Peregoy & Boyle, 1997, p. 93). Therefore, assessment practices should allow students to show what they know in a variety of ways. When creating assessments, consider the diversity of students' cultural, linguistic, and special needs (Peregoy & Boyle, 1997). Research also reports on several teaching practices for supporting equitable teaching and learning of mathematics (Aguirre et al., 2013, 2024; Bartell, Wager, Edwards, Battey, Foote, & Spencer, 2017, Ladson-Billings, 2007; Seda & Brown, 2021). For example, the National Council of Teachers of Mathematics recommends the following research-based, equity-based teaching practices (Aguirre et al., 2024, p. 71–72):

- Going deep with mathematics. Teachers enact high cognitive-demanding tasks that support deep conceptual understanding, procedural fluency, problem-solving, and reasoning.
- Leveraging multiple mathematical competencies. Teachers recognize and position students' various mathematical backgrounds and competencies as a resource for learning and teaching mathematics.
- Affirming mathematics learners' identities. Teachers recognize and support students' mathematical identities because a positive, productive mathematics learner contributes to the mathematics learning. Students' mathematical identities develop when the environment offers multiple mathematical contributions, provides multiple entry points, and promotes various ways that students can participate in mathematics.
- Challenging spaces of marginality. Rather than an independent and isolating experience, teachers embrace student competencies, diminish status, and value multiple mathematical contributions.
- Drawing on multiple resources of knowledge. Teachers recognize and leverage students' knowledge and experiences—mathematical, cultural, linguistic, peer, family, and community—as resources for the classroom. This includes "helping students bridge everyday experiences to learn mathematics, capitalizing on linguistic resources to support mathematics learning, recognizing family or community mathematical practices to support mathematics learning, and finding ways to help students learn and use mathematics to solve authentic problems that affect their lives.

Equity-based teaching practices, such as those suggested by the National Council of Teachers of Mathematics, are enacted through CMP problems. Students' perceptions about mathematics come from the problems and environments in which they are asked to engage. CMP4 problems are sources of classroom activity devoted to the developing mathematical ideas involving multiple access points, multiple strategies, and/or multiple solutions. CMP4 problems provide students with opportunities to elicit genuine interest/wonderment in the context itself, offer access to the problem, anchor instruction to develop understandings, and highlight applications to everyday matters. The STEM problem format promotes student engagement and learning as

they collaborate to design solutions, make conjectures, offer critiques, and communicate their mathematical understandings. Types of CMP problems include card sorts, games, experiments, student thinking, models and visuals, matching, hands-on, technology and manipulatives, data sets, capstone problems, and reflections. The STEM problem format provides teachers with flexibility to carry out equitable practices that help address the individual needs of all students. For more information about the CMP STEM problem format, please see Part 2, Building on the Successes of CMP1, CMP2, and CMP3—What's in CMP4?

Equity-based teaching practices, such as those suggested by the National Council of Teachers of Mathematics, are enacted through problems using CMP's instructional model, Launch—Explore—Summarize. In CMP, equity means that all students can participate substantially throughout the Launch—Explore—Summarize phases of mathematics problems. It is important to note that students may not participate in the same ways (Jackson & Cobb, 2010). Several factors influence how problems play out equitably in classrooms: (1) the mathematical goal of the task (Sleep, 2012); (2) the cultural suppositions inherent in the problem situation (Boaler, 2002; Jackson et al., 2013); (3) situation imagery of the mathematical relationships in the task (Thompson, 1996); (4) type and nature of interactions in small groups (Cobb, 1995; Wood & Yackel, 1990); (5) participation norms and assigned competence (Boaler & Staples, 2008); (6) informal and formal language used (Moschkovich, 2002); and (7) teacher knowledge and beliefs about content, students, and pedagogy (Stein, Remillard, & Smith, 2007). For each problem, the Extended Launch—Explore—Summarize includes questions to ask students at each problem phase are included to help teachers support student learning and support ongoing formative assessment. Actual classroom scenarios and examples of student thinking are included in the Launch, Explore, and Summarize sections to help stimulate teachers' imaginations about what is possible. The Extended Launch—Explore—Summarize, there is support for teachers when making instructional decisions in ways that connect explicitly to the Attending to Individual Learning Needs Framework and the Pedagogical Strategies.

For more teacher strategies for implementing the CMP4 curriculum, please see Guide Aid 2.K, Pedagogical Strategies in Part 2, Building on the Successes of CMP1, CMP2, and CMP3—What's in CMP4?

CMP AND MULTILINGUAL LEARNERS (ENGLISH LEARNERS)

The Attending to Individual Learning Needs Framework helps to provide a curriculum appropriate for multilingual learners. Multilingual learners refer to students who are, or have been, consistently exposed to multiple languages. It includes students known as English Learners, English language learners, dual language learners, heritage language learners, and students who speak varieties of English or indigenous languages.

For each problem in CMP4, there are suggestions for multilingual learners in the Teacher Edition that relate to the components of the Attending to Individual Learning Needs Framework. There are also pedagogical strategies to support teachers when making instructional decisions for multilingual learners. The unit alignment chart contains the mathematical goals, the Arc of Learning™ phase, Now What Do You Know?, and the Emerging Mathematical Ideas. For each problem, this chart indicates where it fits within the Arc of Learning™ Framework. The Arc of Learning™ Framework provides the teachers a guide as to how the mathematical understandings evolve from informal to more sophisticated understandings. The chart also highlights the Now What Do You Know? in the Student Edition that connects the embedded understandings with prior and future knowledge. It provides student-facing questions for students to self-assess and consolidate their learning. The Emerging Mathematical Ideas listed

in the chart bring all of the ideas together by listing the mathematics that students will be using as they answer the Now What Do You Know? In the Teacher Edition, teachers are provided with the answers alongside each problem and mathematical reflection from the Student Edition. The answers discuss how students might think about the problem and include examples of student thinking and strategies, and support teachers in how students access problems, employ different strategies, and reach solutions. Multilingual learners come into the classroom from a variety of countries with a diverse set of experiences. They face the daunting tasks of adjusting to a new home and cultural environment, learning a new language, making new friends, and making sense of the rules, appropriate behaviors, and mechanics of a new school. Multilingual learners are experiencing many losses while simultaneously trying to "fit in" with their new surroundings.

For teachers, working successfully with multilingual learners requires more than just teaching the course content. In order for multilingual learners to achieve academic success, teachers must also support language goals and general learning strategies in the mathematics classroom (Richard-Amato & Snow, 2005). In addition, it is critical to create a friendly, supportive, and predictable classroom community. Some general suggestions for teachers include

- learning about students' home countries, languages, and previous educational experiences;
- valuing students' differences as resources;
- staying connected to families; and
- communicating school norms and expectations clearly and checking assumptions at the door.

Instructing multilingual learners is a "lifelong process of learning, discovery, accepting, and trying" (Carger, 1997, p. 45).

Multilingual learners are often anxious about being in a classroom when they cannot speak the language. Efforts to create a friendly environment that is respectful of students' diverse experiences and sets high expectations for learning will greatly support multilingual learners' opportunities for success. Part of establishing this kind of learning environment includes modifying the ways in which teachers talk with students. Many of the suggestions below work for all students, including multilingual learners.

- Create a classroom community that recognizes and values students' diversity. Every child is born into a culture that socializes them to think in specific ways about things that may be taken for granted as common sense. When left unexamined, some cultural beliefs and practices can interfere with students' success in the classroom. It is important to find out who students are, where they come from, and which languages they speak (Barwell, 2020).
- Keep expectations high and consistent and provide effective feedback. Too often, multilingual learners receive "feedback that relates to personality variables or the neatness of their work rather than to academic quality" (Jackson, 1993, p. 55). Comments should focus on the academic components of students' work. It is important to communicate clearly and specifically to students about how to improve the overall quality of the work they do (Jackson, 1993).
- Slow down and simplify the language used. Consider intonation; avoid using slang, idioms, extraneous words, and long, complex sentences. Repeat key points. Rephrase to promote clarity and understanding. Summarize frequently. Use clear transition markers such as *first*, *next*, and *in conclusion*. Ask clear, succinct, high-level questions (Carrasquillo & Rodriguez, 2002; Jameson, 1998).
- Model what students are expected to do. Students may not comprehend the words or phrases being said, yet actions will support their understanding. For example, use visual prompts such as hand movements, facial expressions, or other body movements to suggest meaning.

- Pair instructional talk with visual communication cues such as pictures, graphs, objects, and gestures (Peregoy & Boyle, 1997; Chval et al., 2021).
- Even though content will vary, follow a predictable routine and a stable schedule. Predictability creates a sense of security for students who are experiencing a lot of change in their lives (Peregoy & Boyle, 1997).
- Have dictionaries and other learning tools available easily accessible to students.

Multilingual learners benefit from a variety of instructional strategies that lower their anxiety and help make content more comprehensible. Mathematical objectives should be cognitively demanding and grade appropriate. Language-related adjustments should be made, including modifications to instructional delivery, but the cognitive demand of the mathematics should not be changed. The CMP curriculum inherently supports many, if not all, of the following strategies for multilingual learners. These strategies are also good teaching techniques for all students. (For more on teaching CMP strategies, see Part 3.)

- **Effective Questioning.** Teachers "frequently use few higher-order questions to all students, especially to those for whom they had low expectations" (Jackson, 1993, p. 55). Higher-order questions promote analytical and evaluative thinking, affirm students' self-perceptions as learners, and support students' view of themselves as knowledge producers rather than knowledge consumers (Jackson, 1993).
- **Collaborative Groups.** Research evidence demonstrates that collaborative group work can have a "strong positive impact on language and literacy development and on achievement in content areas" (Richard-Amato & Snow, 2005, p. 190).
- **Active Participation.** Actively engaging the students facilitates their learning of both mathematics and the English language. This can be accomplished through class brainstorms and predictions and by encouraging students to ask questions of each other or to express and justify their ideas. It is also helpful to write students' ideas on the chalkboard so they can see them written correctly in English.
- **Situational Context.** If students are unfamiliar with names, places, or objects in a problem, it will be difficult for them to access the mathematics. Sometimes it is possible to change the context of a problem without affecting the mathematics or objectives of the lesson. Incorporate names and places from students' home countries, or situate actions within their cultural practices. This is also a great opportunity for students to learn common English words used in daily life. Include words in math problems that students need to know by avoiding the use of slang, idioms, or extraneous language.
- **Written Expression.** Use journals, quick writes, and outlines to provide students with opportunities to write in the mathematics classroom. The following suggestions promote meaningful writing activities: (a) Restate the problem in your own words; (b) Explain how you solved the problem; (c) How do you know your answer is right? (Richard-Amato & Snow, 2005); and (d) What do you know so far about . . .?

Students must understand mathematical terminology and key words to gain access to any math problem. Teachers can support them by doing the following:

- Isolate important key terms and phrases by circling or underlining them in the text.
- Create and maintain a Word Cluster or Key Terms Chart in the classroom and in students' notebooks where new terms and their definitions are written in both English and the students' first language. Pictures are also useful additions.
- Practice speaking hard-to-pronounce words verbally as a class. It is beneficial for students to practice reading and pronouncing words correctly.

- Use graphic organizers to scaffold learning activities and provide multilingual learners access to the mathematical content. Graphic organizers include Venn diagrams, concept webs, timelines, lists, outlines, tree diagrams, and charts.

Finally, the following strategies will enhance multilingual learners' abilities to participate in classroom discussions and to express their ideas in writing assignments.

- **Give Time to Prepare.** Give students time to read silently before asking them to discuss their ideas with a small or large group. It is also recommended to provide time for students to write their ideas on paper before they share them publicly. This will give students time to sort through their ideas before they are asked to perform in front of teachers and peers.
- **Write and Speak Directions.** Post task directions on the overhead or chalkboard while simultaneously reading the directions and having students follow along. This affords multilingual learners the opportunity to read the English text silently while they hear it spoken correctly.
- **Encourage the Use of Comfortable Languages.** Encourage multilingual learners to write and speak in both their home language and English. It is also helpful to ensure that it is okay even if their spelling and grammar is incorrect.
- **Allow Extra Wait Time.** Use extra wait time so multilingual learners will have an opportunity to hear the question, translate the work, understand its content, formulate a response, and then speak.

Students' lack of English proficiency will affect test performance when tests are given only in English. Use a variety of assessments in a variety of formats, including small-group work, individual activities, drawing pictures, creating posters, engaging in interviews, constructing portfolios, journal writing, projects, and self-assessment.

- Be clear and consistent with grading systems and standards. Rubrics are excellent tools for itemizing the criteria on which students will be assessed and helping students understand what is expected from them (Richard-Amato & Snow, 2005).
- Peer editing is an opportunity for students to read, edit, and comment on each other's work while gaining reading and writing experience.
- Often multilingual learners get so bogged down in the reading comprehension that they never get to the mathematics. It will be much more meaningful and productive for the students if they are assigned 5 or 6 well-designed exercises (and they'll be more motivated to try them), rather than a page or two of 10 to 20 exercises.
- Allow sufficient time for all students to complete the assessment.

The following suggestions follow guidelines known as rebus techniques for multilingual learners. *Rebus* is a general term referring to the use of pictures or other visual images to represent words or symbols. Some of these techniques are similar to those in the preceding sections.