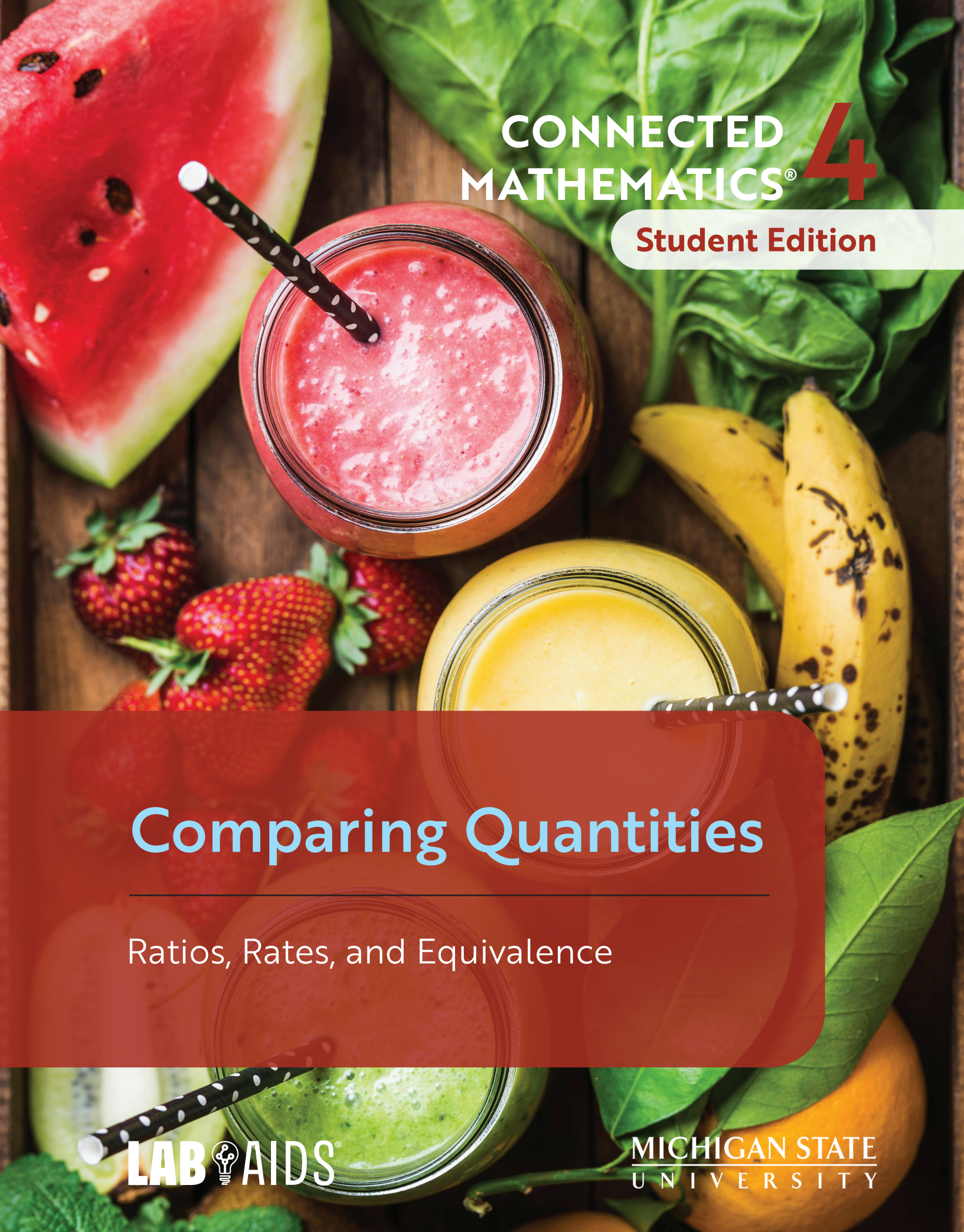

CONNECTED MATHEMATICS® 4

Student Edition

Comparing Quantities

Ratios, Rates, and Equivalence

LAB AIDS®

MICHIGAN STATE UNIVERSITY

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***In memory of* . . . William Fitzgerald (Deceased)** Bill through his making "good trouble" made substantial contributions to conceptualizing and creating *Connected Mathematics*®.

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CONTENTS

MATHEMATICAL GOALS

Reasoning with Ratios. Understand ratios.

- Understand ratios as a multiplicative comparison of two quantities and distinguish them from difference (additive) relationships
- Understand that ratios can be either part-to-part or part-to-whole comparison
- Understand unit rate as a ratio where one of the two quantities being compared has a value of 1
- Understand that a percent is a part-to-whole ratio where the whole is 100
- Recognize and represent equivalent ratios, including percents and unit rates, with ratio tables, tape diagrams, double number lines, and equations
- Identify ratio and rate situations and choose appropriate representations and/or strategies to reason with ratios when solving problems
- Use concepts associated with recognizing, representing, and using relationships between two variables, begun in the *Variables and Patterns* and *Number Connections* units, to work with ratio relationships

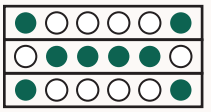
LOOKING AHEAD

Washington students plan to sell green and white cupcakes in large and small boxes.

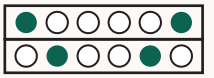
The students at Washington school have 40 white cupcakes to package.

If they put them in large boxes, **how many** green cupcakes will they need?

If they put them in small boxes of cupcakes, **how many** green cupcakes will they need?



Large Box
8 green cupcakes
10 white cupcakes



Small Box
4 green cupcakes
8 white cupcakes

Students decide to do a walking backward race. They do a few trials. They find that the average middle school student can walk $\frac{1}{4}$ of a mile backward in 6 minutes.

The students schedule the race for 30 minutes. Whoever walks the farthest is the winner.

How many miles would the average middle school student cover in this amount of time? Explain your reasoning.

Have you ever tried walking or running backward?

The following data came from a Central Middle School classroom.

Trait	Yes	No	Total
Attached Earlobes	12		30
Dimples	6		30
Straight Hair	24		30
Widow's Peak	18		30

What percent of the students in the Central classroom have each trait?

If this percent is typical for the whole school, **how many** of the 500 students in the school have these traits?

Detached Earlobe

Attached Earlobe

Dimple

No Dimple

Straight Hair

Curly Hair

Widow's Peak

No Widow's Peak

Numbers are useful in solving problems. In the *Number Connections* unit, we used the *multiplicative structure* of number to solve problems. For example:

- When we realize that every number is a product of its factors, like $12 = 1 \times 2 \times 2 \times 3$, we can analyze the locker problem to predict which students closed or opened a locker.
- Least common multiples were used to predict the next time cicadas and their predators would appear together.
- Greatest common factors were useful to determine the number of snack packs to make with equal amounts of snacks in each pack.
- We also used algebraic expressions to express all multiples of 3 as $3N$, all multiples of 5 as $5N$, and all common multiples of 3 and 5 as $15N$, where N is a whole number.

Numbers are also useful for comparing quantities.

- Comparing by subtracting and finding the difference is one way: "I have $2.00 more than my sister."
- Comparing with *ratios* uses multiplication: "I have twice as much money as my sister."

In *Comparing Quantities*, we will use our understanding of the multiplicative structure of number to develop skills with ratios and percent. Our new skills can help us make sense of situations like the ones on the previous page. Solving problems with ratios is called *reasoning with ratios*.

As we work on problems in this unit, ask yourself questions about situations that involve comparisons.

- **Is** this a comparison situation? If so, do I use ratios or subtraction?
- **What** strategies can I use to find equivalent ratios?
- **How** can I use ratios, unit rates, or percent to make comparisons and solve problems?

INVESTIGATION

1

Making Comparisons

People make and do amazing and amusing things all over the world. For instance, the smallest motorized car is so small it has a bumper $\frac{1}{2}$ the thickness of a human hair, and its top speed is 0.011 miles per hour. It's so small it can sit on a fingernail!

The following are statements about people, places, and things. Notice that numbers are used to justify these claims.

- The longest plunge over the edge of a waterfall in a kayak by a woman is 82 feet.
- The region of the world with the most biodiversity is the Tropical Andes of South America, where approximately 16% of all known plant species live.
- The winner of the first official backward running race ran one mile in 6 minutes 2.35 seconds.
- The Department of Commerce claimed that the number of STEM (science, technology, engineering, and mathematics) jobs between 2008 and 2018 grew at twice the rate as other jobs.
- On average, each one of us produces 4.4 pounds of solid waste each day.

Which of these statements make a comparison? What are they comparing?

PROBLEM 1.1

Fundraising: Making Comparisons

Students at Academy, Central, and Washington middle schools are raising funds to support a local charity.

Each school picks a different goal for its fundraiser. The fundraising goal for each school is displayed on a banner outside each school.

INITIAL CHALLENGE

The Academy students wrote some claims about the fundraising goals on slips of paper and gave them to the principal to read over the loudspeaker during the morning announcements.

Markus

The Academy goal is $150 more than the Washington goal.

Tabia

For every $60 the students at Academy plan to raise, the students at Central plan to raise $90.

Kimi

When the students at Academy meet their goal, they will have raised $\frac{2}{3}$ of Central's goal.

Eliza

The Washington goal is 50% of the Academy goal.

Lakisha	Chung
The Washington goal is half the Academy goal.	For every $3 the students at Washington plan to raise, the students at Central plan to raise $1.

Andres	Sharon
For every dollar the students at Washington plan to raise, the students at Academy plan to raise two dollars.	The students at Central will raise the most money.

- Decide whether each student's claim is true. Explain your reasoning.
- Which statements help you compare two of the fundraising goals? Explain why.

WHAT IF . . . ?

Situation A. Students' Observations

The students at Academy were trying to decide which comparison statements in the Initial Challenge to use to advertise their goals. Several students and the principal made observations about the goal statements. Examine each observation and answer the question.

Markus's Observation	Eliza's Observation	Lakisha's Observation
I like the statements that used "for every" and wondered if the other statements could be rewritten as "for every" statements. Is this possible? Explain.	I think Sharon's statement is true, but it is not a comparison statement. Is she correct? Why?	I like Markus's comparison statement because it involves subtraction. Is she correct? Why? Do other comparison statements involve subtraction? Explain.

Principal's Statement

Maybe we should add this statement.

> For every $100 the students at Washington plan to raise, the students at Academy plan to raise $200.

Is this a true statement? Explain.

Andres's Question

I think the principal's statement is the same as mine.

Do you agree with Andres? Explain.

NOW WHAT DO YOU KNOW?

What do you notice about different kinds of comparison statements?