UNIT ALIGNMENTS

GOALS, ARC OF LEARNING™, STANDARDS, NOW WHAT DO YOU KNOW?, AND EMERGING MATHEMATICAL IDEAS

Investigation 1. Making Comparisons

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 1.

Reasoning with Ratios. Understand ratios.

- Understand ratios as a multiplicative comparison of two quantities and distinguish them from difference (additive) relationships
- Understand that ratios can be either part-to-part or part-to-whole comparison
- Understand unit rate as a ratio where one of the two quantities being compared has a value of 1
- Understand that a percent is a part to whole ratio where the whole is 100
- Recognize and represent equivalent ratios, including percents and unit rates, with ratio tables, tape diagrams, double number lines, and equations
- Identify ratio and rate situations and choose appropriate representations and/or strategies to reason with ratios when solving problems
- Use concepts associated with recognizing, representing, and using relationships between two variables, begun in the *Variables and Patterns* and *Number Connections* units, to work with ratio relationships

Arc of Learning™, Standards, Now What Do You Know?, and Emerging Mathematical Ideas

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Problem 1.1. Fundraising: Making Comparisons **Arc of Learning™: Introduction** **Standards:** See your state alignment chart.	
Now What Do You Know? What do you notice about different kinds of comparison statements?	**Emerging Mathematical Ideas** Build on current use of additive relationships in making comparisons to also include comparison statements that employ multiplicative relationships. Students may notice • some comparisons include one number, others two; • comparisons may use fractions, percentages, whole numbers, or "for every" or "for each" language;

	• some comparisons use multiplication, others addition; • they compare two or more values for the same *thing* that may come directly from the context or the values may be scaled; and • they help you think about relative sizes—or relationships between quantities
Problem 1.2. Fundraising Update: Ratios as Comparisons **Arc of Learning™: Introduction, Exploration** **Standards:** See your state alignment chart.	
Now What Do You Know? How do you decide when a comparison statement is a ratio? How do you decide when two ratios describe the same relationship?	**Emerging Mathematical Ideas** Work to begin understanding the mathematical characteristics that define **ratio** comparison statements. Students may notice • ratios are comparisons of two quantities; • ratios describe the multiplicative relationship between the quantities; • ratios can be written with the actual numbers (raw values) or with scaled equivalent values; • language that suggests a ratio comparison may include phrases like "for every" or "for each" or "to"; • ratios are NOT comparisons that describe the difference between two values; and • ratios may use either part-to-part or part-to-whole comparisons.
Problem 1.3. Making Cupcakes: Part-to-Part or Part-to-Whole Ratios **Arc of Learning™: Exploration** **Standards:** See your state alignment chart.	
Now What Do You Know? How can you decide when a ratio is comparing a part to a part? A part to a whole? How can you decide when two ratios are equivalent?	**Emerging Mathematical Ideas** Deepen understanding and recognition of various forms for **ratio** comparison statements. Ratio comparisons can be • written as part-to-part and as part-to-whole comparisons; • equivalent, look different, and describe the same multiplicative relationship between two quantities; and • written with either of the two quantities coming first as long as the quantities are labeled correctly.
Mathematical Reflection **Arc of Learning™: Exploration** **Standards:** See your state alignment chart.	
Mathematical Reflection What do you understand about ratios and reasoning with ratios?	**Emerging Mathematical Ideas** Early exploration of ratios and equivalent ratios as a comparison form used when two quantities have a multiplicative relationship. Begin to use and articulate references to the following ratio concepts: • *Comparison statements* compare two quantities (fractions, percents, whole numbers) using language such as "for every" or "for each" or "to." • Comparison statements can reference *additive* or *multiplicative* relationships between two quantities. • *Ratios* are a comparison of two quantities with a multiplicative relationship. • Ratios can be written with the actual numbers (raw numbers) or with an *equivalent* (scaled) relationship between the quantities.

	• Ratios can be written as *part-to-part* or *part-to-whole* comparisons. • Ratios with the same multiplicative relationship are *equivalent ratios*. • The order in which quantities are compared in a ratio statement does not matter so long as the quantities are labeled correctly. Note that relevant developing vocabulary in this investigation has been *italicized*.

Investigation 2: Using Ratios to Solve Problems

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 2.

Reasoning with Ratios. Understand ratios.

- Understand ratios as a multiplicative comparison of two quantities and distinguish them from difference (additive) relationships
- Understand that ratios can be either part-to-part or part-to-whole comparison
- Understand unit rate as a ratio where one of the two quantities being compared has a value of 1
- Understand that a percent is a part-to-whole ratio where the whole is 100
- Recognize and represent equivalent ratios, including percents and unit rates, with ratio tables, tape diagrams, double number lines, and equations
- Identify ratio and rate situations and choose appropriate representations and/or strategies to reason with ratios when solving problems
- Use concepts associated with recognizing, representing, and using relationships between two variables, begun in the *Variables and Patterns* and *Number Connections* units, to work with ratio relationships

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Problem 2.1. Packaging Cupcakes: Using Ratios **Arc of Learning™: Exploration** **Standards:** See your state alignment chart.	
Now What Do You Know? How are equivalent ratios useful in solving this problem?	**Emerging Mathematical Ideas** Continue exploring ratios with a focus on finding and using equivalent ratios to solve problems. Begin to notice, use, and articulate that • equivalent ratios are related by a common multiplicative factor; • equivalent ratios can be found by scaling up (multiplying) a given ratio or scaling-down (dividing) from the given ratio; and

	• using ratio relationships to solve problems is referred to as reasoning with ratios and can be a quick and powerful tool for problem solving. Some may *begin* to notice that identifying a unit rate, 1:*x* or *y*:1, can be useful for solving problems with ratios.
Problem 2.2. Selling Cupcakes: Representing Ratios with Tables **Arc of Learning™: Exploration** **Standards:** See your state alignment chart.	
Now What Do You Know? How can a table of equivalent ratios be useful in solving problems with ratios? What information is added by looking at equations or graphs that represent ratios?	**Emerging Mathematical Ideas** Begin to organize equivalent ratios into ratio tables, looking for patterns and underlying structure across the equivalent ratios for two quantities of the form $y = ax$. Begin to use ratios associated with a context to • make, organize, and extend ratio tables with equivalent ratios; • use ratio tables to solve problems using values in the table and by extending the table when needed; • make early attempts to describe how ratio tables can be useful in solving ratio problems, including identifying unit rates, 1:*x* or *y*:1. Begin using the relationship between two quantities given in a ratio table to • write an equation of the form $y = ax$ representing the relationship; and • graph points on the table. Explore connections between the ratio table, an equation, and the graph each representing the same relationship between to variables.
Problem 2.3. Sharing Smoothie Bars: More Ratios **Arc of Learning™: Exploration, Analysis** **Standards:** See your state alignment chart.	
Now What Do You Know? How can visual representations, like the bars that represent smoothie bars, help you think about equivalent ratios? How are these bars useful in solving problems?	**Emerging Mathematical Ideas** Introduce the use of tape diagrams via smoothie bars for solving ratio tasks by creating equivalent ratios and ratio reasoning. Begin to notice • the smoothie bars (tape diagrams) make it easy to visualize the multiplicative relationship described by a ratio; • the smoothie bar partitions can be further partitioned when more pieces are needed to find equivalent ratios with factional parts of a segment (like Sofia and Dembe in the Initial Challenge); • the smoothie bars can be easily duplicated to create additional equivalent ratios; and • the connections between patterns found in ratio tables for equivalent ratios and the concrete representations of the ratios using the smoothie bars.
Mathematical Reflection **Arc of Learning™: Exploration, Analysis** **Standards:** See your state alignment chart.	
Mathematical Reflection What do you understand about ratios and reasoning with ratios?	**Emerging Mathematical Ideas** Begin to notice, use, and articulate that • equivalent ratios are related by a common multiplicative factor; • equivalent ratios can be found by scaling up (multiplying) a given ratio or scaling down (dividing) from the given ratio;

	• using ratio relationships to solve problems is referred to as reasoning with ratios and can be a quick and powerful tool for problem solving; and • making a ratio table helps to organize equivalent ratios, is a useful tool for problem solving with ratios, and is a helpful in looking for patterns among the equivalent ratios.

Investigation 3: Using Unit Rates and Rate Tables

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 3.

Reasoning with Ratios. Understand ratios.

- Understand ratios as a multiplicative comparison of two quantities and distinguish them from difference (additive) relationships
- Understand that ratios can be either part-to-part or part-to-whole comparison
- Understand unit rate as a ratio where one of the two quantities being compared has a value of 1
- Understand that a percent is a part-to-whole ratio where the whole is 100
- Recognize and represent equivalent ratios, including percents and unit rates, with ratio tables, tape diagrams, double number lines, and equations
- Identify ratio and rate situations and choose appropriate representations and/or strategies to reason with ratios when solving problems
- Use concepts associated with recognizing, representing, and using relationships between two variables, begun in the *Variables and Patterns* and *Number Connections* units, to work with ratio relationships

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Problem 3.1. Selling Smoothie Bars: Unit Rates **Arc of Learning™: Exploration, Analysis** **Standards:** See your state alignment chart.	
Now What Do You Know? How are rate tables and unit rates and equations useful in solving problems?	**Emerging Mathematical Ideas** Begin recognizing and making sense of unit rates in various representations and explore using them to solve ratio problems. Notice that • one of the quantities in a unit rate comparison is a 1, *y*:1 or 1:*x*; • unit rates are useful in solving problems with ratios because they allow you to easily calculate any quantity in a ratio given the other quantity; and

	• unit rates are useful in building or completing a rate table and in finding other equivalent ratios. Make connections between unit rates and equations relating the two quantities and a graph of the relationship between the two quantities.
Problem 3.2. Making Popcorn: More Unit Rates and Tables **Arc of Learning™: Exploration, Analysis** **Standards:** See your state alignment chart.	
Now What Do You Know? What are the advantages of using rate tables and unit rates to solve problems? How might an algebraic expression or equation be useful?	**Emerging Mathematical Ideas** Continue building expertise in working with unit rates to solve ratio problems. For a given ratio relating two quantities, work to: • identify both unit rates given an equivalent ratio; • use the unit rate to complete/extend a ratio table; • use the unit rate to calculate any quantity in a ratio given the other quantity; • identify which of the two unit rates can be used most directly to solve a problem; and • use a unit rate to write an algebraic expression that can be used to calculate one of the ratio values given the other value. Begin to notice and articulate that when looking for a specific value in a ratio relationship, there can be an advantage to using a unit rate and/or an algebraic expression over a ratio table because the table may or may not have the values needed. Unit rates and algebraic expressions allow you to calculate needed values directly.
Problem 3.3. Experimenting with Slime: Solving Problems with Ratios **Arc of Learning™: Exploration, Analysis** **Standards:** See your state alignment chart.	
Now What Do You Know? What strategies did you use to solve problems that involve ratios and unit rates?	**Emerging Mathematical Ideas** Continue building expertise in working with unit rates to solve ratio problems of greater complexity. Work to find needed equivalent ratios using a variety of strategies, including • unit rates; • ratio tables; • multiplying/dividing both quantities in a ratio (scaling) to find larger or smaller values as needed; and • algebraic expressions to generate equivalent ratios.
Problem 3.4. Walking Backward: More Ratio Problems **Arc of Learning™: Analysis** **Standards:** See your state alignment chart.	
Now What do You Know? What strategies did you use to solve this problem? How do they compare to strategies you used in previous problem?	**Emerging Mathematical Ideas** Continue building expertise in working with unit rates to solve ratio problems of greater complexity. Work to find needed equivalent ratios using a variety of strategies, including: • unit rates; • ratio tables; • multiplying/dividing both quantities in a ratio (scaling) to find larger or smaller values as needed;

	• expressions (not asked for in Problem 3.4); and • other informal strategies based on multiplicative reasoning and number sense needed to compose and decompose given values to find needed values (Initial Challenge).
Mathematical Reflection **Arc of Learning™: Analysis** **Standards:** See your state alignment chart.	
Mathematical Reflection What do you understand about ratios and reasoning with ratios?	**Emerging Mathematical Ideas** Understand and use ratio concepts. Make connections among rates, equivalent rates, unit rates, and their representations in ratio tables, expressions, equations, and graphs. Make sense of and use an expanding collection of strategies for finding equivalent ratios and for solving ratio problems, including • unit rates; • ratio tables; • multiplying/dividing both quantities in a ratio (scaling) to find larger or smaller values as needed; • apply algebraic expressions to generate equivalent ratios; • other informal strategies based on multiplicative reasoning and number sense needed to compose and decompose given values to find needed values (Initial Challenge); and • diagrams to represent the two quantities in the ratio.

Investigation 4: For Every 100: Introducing Percents

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 4.

Reasoning with Ratios. Understand ratio.

- Understand ratios as a multiplicative comparison of two quantities and distinguish them from difference (additive) relationships
- Understand that ratios can be either part-to-part or part-to-whole comparison
- Understand unit rate as a ratio where one of the two quantities being compared has a value of 1
- Understand that a percent is a part-to-whole ratio where the whole is 100
- Recognize and represent equivalent ratios, including percents and unit rates, with ratio tables, tape diagrams, double number lines, and equations
- Identify ratio and rate situations and choose appropriate representations and/or strategies to reason with ratios when solving problems
- Use concepts associated with recognizing, representing, and using relationships between two variables, begun in the *Variables and Patterns* and *Number Connections* units, to work with ratio relationships

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<table>
<tr><td colspan="2">Problem 4.1. Percent: Out of 100
Arc of Learning™: Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What do You Know?
What is a percent? How is percent like a ratio?</td><td>Emerging Mathematical Ideas
Begin making sense of percents by connecting to earlier part-to-whole ratio comparison statements, with the distinction that this time the whole is 100.
Notice that
• percents are part-to-whole ratios that always compare to a total of 100;
• percents tell you how many out of 100;
• 10 × 10 grids and bar grids are useful representations for percents; and
• percent ratios make comparing ratios quick and easy.</td></tr>
<tr><td colspan="2">Problem 4.2. Surveys, Percents, and Tape Diagrams
Arc of Learning™: Exploration, Analysis
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What do You Know?
How are diagrams such as percent bars (tape diagrams) or double number lines helpful for answering percent questions? What other strategies might be helpful?</td><td>Emerging Mathematical Ideas
Make connections between and among a variety of strategies for solving problems involving percents.
Look closely at a variety of percent problem-solving strategies to notice that
• tape diagrams and double number lines allow you to easily visualize the size of a percentage;
• ratio tables, unit rates, scaling, and 10 × 10 area grids can each be useful in solving some rate problems; and
• while many strategies may be used to solve percent problems, some may be more or less efficiently applied than others and all strategies should be considered.
Solve a variety of percent problems, including those asking students to find
• the percent given a ratio;
• the quantity in the whole given the ratio and amount in the part of the whole; and
• the quantity that is the part of the whole given the percent and amount in the whole.</td></tr>
<tr><td colspan="2">Problem 4.3. Genetic Traits: Finding Any Percent
Arc of Learning™: Exploration, Analysis
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What do You Know?
How does finding 1% help you scale to another percent? How is using a percent bar like using a rate table?</td><td>Emerging Mathematical Ideas
Continue building expertise in applying a variety of strategies to solving percent problems of increasing complexity.
Apply percent representations and strategies to new contexts with opportunities to
• make sense of and use information given in the form of a tape diagram;</td></tr>
</table>

	• connect the use of unit rates from rate tables with percent calculations scaling up from 1%; and • use multiple strategies to solve percent problems.
Problem 4.4. Making Sense of Percent Situations **Arc of Learning™: Exploration, Analysis** **Standards:** See your state alignment chart.	
Now What do You Know? How is percent used when making comparison statements? How are these the same as or different from other comparison statements?	**Emerging Mathematical Ideas** Continue building expertise in applying multiple strategies to a variety of percent problems with increasing complexity. Appreciate and notice • the power of percent ratios when comparing quantities relative to the total; • percents let you compare ratios by finding equivalent ratios for quantities out of 100; and • percents *cannot* be used to directly compare two quantities if they are not part of the same whole.
Mathematical Reflection **Arc of Learning™: Analysis** **Standards:** See your state alignment chart.	
Mathematical Reflection What do you understand about ratios and reasoning with ratios?	**Emerging Mathematical Ideas** May have noticed • percents are part-to-whole ratio comparisons with the form x:100; • percents are useful for solving ratio problems because they are a convenient way to compare two ratios with different wholes; • diagrams (tape and area models) are helpful to visualize how large a percentage is; • a percent bar tells how much of a quantity is in 100% and can be used to determine how much is in 1%; • knowing how much 1% is can be used to find other percents by multiplying; • percents are used to compare values relative to a total amount (the whole); and • percents *cannot* be used to directly compare two quantities if they are not part of the same whole.