

UNIT DESCRIPTION

This unit is devoted to an introduction to ratios and reasoning with ratios, also called proportional reasoning. A *ratio* is a comparison of two quantities that depends on multiplication and division. A *difference*, by contrast, is a comparison of two quantities that depends on addition and subtraction. The latter does not represent a ratio. Mathematically, students typically come to grade 6 thinking additively, and they tend to compare using differences. In this unit, they will learn that comparing with ratios is better suited for certain situations. This work builds upon the concepts in *Number Connections* and equivalent fractions. Equivalence of ratios is introduced in Investigation 1 and continues throughout the unit. The students use rate tables as a way to express equivalent ratios and compute unit rates. In grade 7, equivalent ratios are called proportional relationships.

Working with equivalent ratios is similar to working with equivalent fractions. For most of this unit, however, ratios are not written as fractions. The intent is to keep the notation for part-whole fractions and rational numbers apart from the notation for ratio comparisons. It will help students to keep these ideas separate as they develop understanding of ratio. When the word *fraction* appears, it is used to represent part of a whole. Equivalent ratios can be represented in tables, tape diagrams, double number lines, grids, and bars. Percent is introduced in Investigation 4 as a ratio. The main goal of this unit is to begin the process of recognizing a ratio situation and then applying an appropriate strategy for solving the problem.

The development of algebraic reasoning continues in this unit as an opportunity to generalize patterns that occur in ratio tables. This continues the theme that algebraic expressions and equations represent important number properties and relationships among quantities.

Reasoning with ratios is revisited and deepened throughout grade 7 as a context to study more complex numeric situations, similarity (scale drawings), linearity, probability, and data. In grade 7, reasoning with ratios becomes proportional reasoning, which involves proportional relationships.

As with all of the *Connected Mathematics*® 4 units, one Mathematical Reflection guides the development of the understanding of the mathematical ideas in the unit.

Mathematical Reflection

In this unit, we are exploring problems that involve reasoning with ratios. At the end of this investigation, ask yourself:

What do you understand about ratios and reasoning with ratios?

SUMMARY OF INVESTIGATIONS

Investigation 1: Making Comparisons

Students explore ratios in the mathematical context of understanding and making comparison statements. Understanding ratios and equivalence of ratios is a major emphasis. In this unit, ratios are comparisons between numbers. They use visual models, symbols, and language to express ratio comparisons. The measuring of progress in a school fundraiser focuses students' attention on ratios as they compare the goals and progress of different grades. A comparison statement that represents a multiplicative relationship between the two quantities is a ratio. These statements can be represented by "for every" statements. For example, for every \$3 the Washington students plan to raise, the Central students plan to raise \$1. Problem 1.1 uses comparison statements about fundraising goals of three schools to introduce multiple ways to describe a situation. In Problem 1.2, students focus on ratios as comparisons using the context of student preferences for pizza. Selling cupcakes as a fundraiser is the context for Problem 1.3 to develop part-to-part or part-to-whole ratios.

Investigation 2: Using Ratios to Solve Problems

Using what they know about comparison statements and equivalent fractions, students develop strategies for finding equivalent ratios. In Problem 2.1, the context of making and packaging cupcakes provides an opportunity to look at comparison statements that involve part-to-part comparisons and part-to-whole comparisons. For example, for every 8 green cupcakes, there are 10 white cupcakes is a part-to-part comparison. For every 20 cupcakes, there are 8 green cupcakes is a part-to-whole comparison. In Problem 2.2, students create equivalent ratios by scaling up or down by multiplying each quantity by the same number (scale factor). Ratio tables are used to represent equivalent ratios. These tables allow students to look for general patterns and express these patterns in words and algebraic expressions. Sharing smoothie bars provides a visual model similar to a tape diagram or bar to find and represent equivalent ratios in Problem 2.3. This allows students to learn about partitioning while keeping track of the relationships among shares of segments of a bar. Students write equivalent ratios to represent the partitioning.

Investigation 3: Using Unit Rates and Rate Tables

Unit rates are introduced in Problem 3.1 as a specific kind of ratio in which one quantity being compared is 1 unit. Rate tables are an extension of ratio tables and are generally used when the two quantities being compared have different units of measurement. For example, for every 6 boxes of smoothie bars, it costs \$48. The unit rate for 1 box is \$8. This can be found by scaling the original ratio/rate down to a unit rate. Rate tables and visual models are important

strategies for developing understanding of ratios and unit rates. Problem 3.2 has students encounter two-unit rates for a situation using the context of the relationship between the number of ounces of popcorn kernels and number of cups of popcorn. Problem 3.3 uses a recipe to make slime to continue looking at scaling quantities up or down. The setting of a walking backward race in Problem 3.4 gives students the setting to refine their strategies for using ratios.

Investigation 4: For Every 100: Introducing Percent _____

Percent is introduced as a comparison statement in which one of the quantities is 100. For every 100 students, there are 30 sixth graders. This can also be written as 30 out of 100 students are sixth graders or 30% of the students are sixth graders. In Problem 4.1, students look at survey data to make comparisons using percents. The percent bar, tape diagrams, and tables are used to visualize and analyze situations involving ratios. They are also useful for relating ratios, fractions, and percents. Problem 4.2 uses a percent bar or tape diagram to analyze survey data. Students use percents to make comparisons among the frequencies of various genetic traits in a class of sixth-grade students in Problem 4.3. In Problem 4.4, using knowledge about a percentage of a given total, students also make predictions using the strategies listed above. They also use these tools to find the whole given the percent and a part.