## UNIT DESCRIPTION

Knowledge of similarity is important to the development of students' understanding of the geometry in their environment. In their immediate environment and in their studies of natural and social sciences, students frequently encounter phenomena that require familiarity with the ideas of enlargement, scale factors, area growth, indirect measurement, and other similarity-related concepts. Similar figures are another name for **scale drawings.** In scale drawings, the original figure and its image are similar figures. In this unit, we will use the two terms interchangeably.

Similarity is also a context in which students' understanding of proportional reasoning continues to deepen. For example, if you increase the size of a diagram by 50%, then distances in the enlarged diagram are proportional to distances in the original diagram. Specifically, every distance in the enlargement is a constant multiple (1.5) of the corresponding distance in the original. It is generally recognized that understanding proportional reasoning is an important stage in cognitive development.

Students in the middle grades often experience difficulty with ideas of scale, which is the heart of proportionality. They confuse adding situations with multiplying situations. Situations requiring comparison by addition or subtraction come first in students' experience with mathematics and often dominate their thinking about any comparison situation, even those in which scale is the fundamental issue. For example, when considering the dimensions of a rectangle that began as 3 units by 5 units and was enlarged to a similar rectangle with a short side of 6 units, many students will say the long side is now 8 units rather than 10 units. They add 3 units to the 5 units rather than multiply the 5 units by 2, the scale factor. These students may struggle to build a useful conception that will help them distinguish between situations that are additive and those that are multiplicative (calling for scaling up or down).

The problems in this unit are designed to help students begin to accumulate the knowledge and experiences necessary to make these kinds of distinctions and to reason about scaling in geometry situations. The next unit, *Comparing and Scaling,* will continue to deepen their reasoning with proportions in the context of quantities. It develops these ideas in numerical, rather than geometric, contexts. Students develop understanding of what it means to be similar figures (scale drawings). This knowledge is required for students to be successful in the grade 8 unit *Flip, Spin, Slide, and Stretch.*

As with all the *Connected Mathematics*® 4 units, one Mathematical Reflection guides the development of the understanding of the mathematical ideas in the unit.

**Mathematical Reflection**

In this unit, we used proportional reasoning to solve problems involving similar figures (scale drawings). At the end of this investigation, ask yourself:

What do you know about similar figures? What do you know about proportional reasoning?

# SUMMARY OF INVESTIGATIONS

## Investigation 1: Enlarging and Reducing Shapes

This investigation informally introduces ideas about mathematical similarity by focusing on scale drawings. Students explore mathematical similarity by relating it to the use of the word *similar* in everyday life.

In Problem 1.1, students estimate the height of a mystery person in a picture by taking an actual measurement into account.

In Problem 1.2, students use rubber bands to enlarge a figure. Students compare how the general shape, lengths of line segments, areas, perimeters, and angles are affected.

In Problem 1.3, students investigate how a copy machine can produce similar figures both larger and smaller than the original. They use percents to compare the scale drawings.

Students explore similar figures in different sizes and compare changes in their side lengths, angle measures, perimeters, and areas. They determine which properties of a figure change and which properties remain the same when a figure is scaled up and down.

## Investigation 2: The Mug Wump Family: Similar Figures

Students build a good working definition of *similarity* in mathematical terms. Using the coordinate system, they draw several geometric figures representing characters called "Wumps." They begin to see connections between geometry and algebra.

In Problem 2.1, students make similar and non-similar shapes using a coordinate system. Students graph characters called "Wumps" and find that Wump family members are similar to one another. They explore algebraic rules that cause images to change size. They also compare angle measures and lengths of corresponding sides informally as they investigate transformations.

In Problem 2.2, students explore algebraic rules that cause images to change size and to move about the coordinate plane. Students recognize the role that multiplication plays in scaled drawings, laying a foundation for students' later work on proportional relationships in the *Comparing and Scaling* unit.

In Problem 2.3, students are introduced to the term **scale factor**. Students find that for two figures to be similar, corresponding angles must be congruent and corresponding sides must grow or shrink by the same factor, called the scale factor. They begin to recognize the role multiplication plays in similarity relationships.

## Investigation 3: Scaling Perimeter and Area

During Investigation 3, students deepen their understanding of what it means for two figures to be similar.

In Problem 3.1, students experiment with rep-tiles, shapes where copies are put together to make larger similar figures. Using rep-tiles, students explore the relationship between the areas of two similar figures. Students sometimes have difficulty grasping the idea that area does not grow at the same rate as side length when a figure is enlarged. Experiences with rep-tiles help them build mental images to support their evolving ideas about the relationship between scale factor and area.

In Problem 3.2, students use similarity and scale factors to find unknown measures or measures of distances that cannot be easily measured directly.

In Problem 3.3, students use similar triangles to measure distances on the ground that cannot be measured directly.

## Investigation 4: Similar Figures and Ratios

Investigation 4 links scaled drawings to proportional reasoning.

In Problem 4.1, students use ratios to determine whether or not two figures are similar. They find ratios of two lengths within each figure and then compare those ratios. **Proportions** are introduced as equations stating that two ratios are equal.

In Problem 4.2, students use ratios and scale factors to find missing side lengths of similar figures. In grade 6, students expressed ratios as comparison statements. In this investigation, students extend their use of ratios by expressing ratios as fractions.

In Problem 4.3, students apply their knowledge of similarity to real-world problems. They use the shadow method to find heights of tall objects.