UNIT ALIGNMENTS

GOALS, ARC OF LEARNING™, STANDARDS, NOW WHAT DO YOU KNOW?, AND EMERGING MATHEMATICAL IDEAS

Investigation 1: Exponential Growth Patterns

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 1.

Exponential Functions. Explore problem situations in which two or more variables have an exponential relationship to each other.

- Identify, compare, and solve problem situations that can be modeled with an exponential function (nonlinear) or linear functions using a table, graph, or equation
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation and make connections among the representations and to representations of linear functions
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function

Properties of Exponents. Develop understanding of equivalent exponential expressions.

- Develop properties for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation that represent the dependent variable in an exponential function
- Write and interpret equivalent expressions using the properties for exponents and operations
- Recognize the role and use properties of exponents and scientific notation to solve problems that involve exponential functions

Arc of Learning™, Standards, Now What Do You Know?, and Emerging Mathematical Ideas

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<table>
<tr><td colspan="2">Problem 1.1. Making Ballots: Linear or Nonlinear Relationship?
Arc of Learning™: Introduction, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
What are the variables in this situation? How are they related? How is this relationship shown in a table, graph, and equation?</td><td>Emerging Mathematical Ideas
Identify variables in a problem.
Begin to make conjectures about a nonlinear relationship between two variables as represented in tables and graphs and to consider what an equation might involve.
Connect to prior work on identifying variables and looking for patterns of change in tables and graphs and generalizing patterns as equations (Variables and Patterns, Moving Straight Ahead).</td></tr>
<tr><td colspan="2">Problem 1.2. The King's Reward: Representing Exponential Functions
Arc of Learning™: Exploration, Introduction
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
In what ways are the exponential functions represented in this problem similar to the exponential function in Problem 1.1? How are they different? How do exponential functions compare to linear functions?</td><td>Emerging Mathematical Ideas
Explore the differences between exponential and linear patterns in tables, graphs, and equations, and begin to notice that exponential functions have multiplicative growth while linear functions have additive growth.
Begin to recognize exponential growth patterns in tables, graphs, and equations. Begin to recognize and identify the growth factor.</td></tr>
<tr><td colspan="2">Problem 1.3. Killer Plant Strikes Lake Victoria: y-Intercepts Other Than 1
Arc of Learning™: Exploration, Analysis, Introduction, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
What information do you need to write an equation that represents an exponential function? How is the information represented by the equation represented in a table and graph of the equation?</td><td>Emerging Mathematical Ideas
Recognize and represent exponential growth patterns in a table, in a graph, and with an equation. Notice exponential functions have multiplicative growth.
Explore y-intercept greater than 1 in exponential contexts and the impact of the starting value on the table, graph, and equation.
Make sense of the idea of any number with an exponent of 0.</td></tr>
</table>

<table>
<tr><td colspan="2">Problem 1.4. Growing Mold: Interpreting Equations for Exponential Functions
Arc of Learning™: Exploration, Analysis, Introduction, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How are the growth factor and initial population for an exponential function represented in an equation that represents the function?</td><td>Emerging Mathematical Ideas
Recognize when situations are models of exponential growth. Identify growth factor in a context, table, graph, and equation:
• Context—dependent variable increasing
• Table—increasing by multiplying by a certain number > 1
• Graph—data points are lying on an increasing curve as the independent variable increases, eventually becoming almost vertical.
• Equation—the growth factor is > 1, and if the initial value is greater than 1, it is a factor in the equation.
Identify initial value/population in an exponential situation.
Write exponential functions using the general form $y = a(b^x)$, where a is the initial value and b is the growth factor.</td></tr>
<tr><td colspan="2">Mathematical Reflection
Arc of Learning™: Exploration, Analysis, Introduction, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Mathematical Reflection
What do you know about exponential functions and exponential expressions? How do exponential functions compare to linear functions?</td><td>Emerging Mathematical Ideas
Identify, compare, and solve problems from a variety of different subject areas that represent an exponential growth function (nonlinear) using a table, a graph, and an equation.
Articulate how exponential functions are different from linear functions. Notice that entries in tables or graphs of exponential relationships are not changing by an additive pattern but rather a multiplicative pattern.
Recognize that expressions with exponents represent multiplicative patterns where you have a repeated factor.
Identify the growth factor and initial value in exponential situations, and be able to represent exponential functions using the general form $y = a(b^x)$, where a is the initial value and b is the growth factor.</td></tr>
</table>

Investigation 2: Growth Factors and Growth Rates

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 2.

Exponential Functions. Explore problem situations in which two or more variables have an exponential relationship to each other.

- Identify, compare, and solve problem situations that can be modeled with an exponential function (nonlinear) or linear functions using a table, graph, or equation
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation and make connections among the representations and to representations of linear functions
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function

Properties of Exponents. Develop understanding of equivalent exponential expressions.

- Develop properties for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation that represent the dependent variable in an exponential function
- Write and interpret equivalent expressions using the properties for exponents and operations
- Recognize the role and use properties of exponents and scientific notation to solve problems that involve exponential functions

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<table>
<tr><td colspan="2">Problem 2.1. Reproducing Rabbits: Fractional Growth Rates
Arc of Learning™: Analysis, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How is the growth factor in this problem similar to that in the previous problems? How is it different?</td><td>Emerging Mathematical Ideas
Explore exponential relationships in which the growth factor is not a whole number.
Begin to recognize non-whole-number exponential growth patterns in tables, graphs, and equations. Explore how division can be used to determine growth factor.</td></tr>
</table>

<table>
<tr><td colspan="2">Problem 2.2. Investing in the Future: Growth Rates
Arc of Learning™: Analysis, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How are the growth factor and growth rate for an exponential function related?</td><td>Emerging Mathematical Ideas
Continue to explore exponential relationships, including those in which the growth factor is not a whole number.
Explore the idea of growth rate and how growth rate and growth factor are related. Begin to recognize growth rate
• as a portion of a growth factor;
• can be found by subtracting 1 from the growth factor; and
• is often written as a percent.
Continue to explore the difference between exponential and linear patterns in tables, graphs, and equations and to identify when a situation is linear or exponential and articulate reasons why.</td></tr>
<tr><td colspan="2">Problem 2.3. Making a Difference: Connecting Initial Values to Growth Factors
Arc of Learning™: Analysis, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How does the initial population (value) affect the growth patterns in an exponential function?</td><td>Emerging Mathematical Ideas
Introduce and explore the idea of “compound growth.”
Further understanding of the power and complexity of exponential growth and how the initial value does not impact the growth factor. Also, how growth rates do impact the value each year.
Be able to explain how initial value, growth factor, and growth rate are represented in tables, graphs, and equations.</td></tr>
<tr><td colspan="2">Mathematical Reflection
Arc of Learning™: Analysis, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Mathematical Reflection
What do you know about exponential functions and exponential expressions? How do exponential functions compare to linear functions?</td><td>Emerging Mathematical Ideas
Articulate when situations presented in a context, table, graph, and/or equation represent an exponential growth situation.
Recognize that expressions with exponents represent multiplicative patterns where you have a repeated factor.
Articulate how exponential functions are different from linear functions, noting how entries in a table or graph are changing not by an additive pattern but rather by a multiplicative pattern.
Identify the growth factor and initial value in exponential situations, and be able to represent exponential situations using the form $y = a(b^x)$, where a is the initial value and b is the growth factor.</td></tr>
</table>

Investigation 3: Exponential Decay

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 3.

Exponential Functions (Nonlinear Functions). Explore problem situations in which two or more variables have an exponential relationship to each other.

- Identify, compare, and solve problem situations that can be modeled with an exponential function (nonlinear) or linear functions using a table, graph, or equation.
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation and make connections among the representations and to representations of linear functions.
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function

Properties of Exponents. Develop understanding of equivalent exponential expressions.

- Develop properties for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation that represent the dependent variable in an exponential function.
- Write and interpret equivalent expressions using the properties for exponents and operations
- Recognize the role and use properties of exponents and scientific notation to solve problems that involve exponential functions

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<table>
<tr><td colspan="2">Problem 3.1. Making Smaller Ballots: Introducing Exponential Decay
Arc of Learning™: Analysis, Synthesis, Exploration, Analysis
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How does the pattern of change in this situation compare to the growth patterns you have studied in previous problems? How is the pattern represented in a table, graph, and equation?</td><td>Emerging Mathematical Ideas
Begin to make conjectures about a nonlinear relationship between two variables that is decreasing as represented in tables and graphs. Consider what an equation for such a relationship might involve.
Describe the decreasing pattern as it shows up in table and graphs.
Connect to prior work on inverse relationships and exponential functions that are growing. Notice how one can derive each value from the previous value. (Thinking with Mathematical Models and Investigations 1 and 2 of this unit).</td></tr>
</table>

Problem 3.2. Fighting Fleas: Representing Exponential Decay

Arc of Learning™: Analysis, Synthesis, Exploration, Analysis

Standards: See your state alignment chart.

Now What Do You Know?	**Emerging Mathematical Ideas**
How can you recognize an exponential decay function from a written description? Table? Graph? Equation?	Recognize and represent when situations are models of exponential decay. Identify decay factors in a context, table, graph, and equation: • Context—dependent variable decreasing • Table—decreasing by multiplying a certain number • Graph—data points are lying on a decreasing curve as the independent variable increases, eventually becoming almost horizontal. • Equation—the decay factor is less than 1 but greater than 0, and if the initial value is > 1, it is a factor in the equation.

Problem 3.3. Cooling Water Experiment: Modeling Exponential Decay

Arc of Learning™: Analysis, Synthesis, Exploration, Analysis

Standards: See your state alignment chart.

Now What Do You Know?	**Emerging Mathematical Ideas**
What do you know about exponential decay functions? How is this information represented in a table, graph, and equation? How do they compare to exponential growth functions?	Carry out an experiment and analyze the results. Recognize and represent when situations are models of exponential decay. Identify decay factors in a context, table, graph, and equation: • Context—dependent variable decreasing • Table—decreasing by multiplying a certain number • Graph—data points are lying on a decreasing curve as the independent variable increases, eventually becoming almost horizontal. • Equation—the decay factor is less than 1 but greater than 0, and if the initial value is > 1, it is a factor in the equation.

Problem 3.4. Wrapping Up: Exponential Functions

Arc of Learning™: Synthesis, Exploration

Standards: See your state alignment chart.

Now What Do You Know?	**Emerging Mathematical Ideas**
What are the effects of a and b on the graph of $y = a(b^x)$, $b \neq 0$? Compare those effects to the effects of m and b in the linear equation $y = mx + b$.	Explore the effects of a and b on the graphs of exponential functions ($y = a(b^x)$, $b \neq 0$). Notice patterns in exponential general equation form and the graph: • If $b > 1$, then the graph is an increasing curve. The greater the value of b then the faster the increase is in the *y value*. • If $0 < b < 1$, then the graph is a decreasing curve. The greater the value of b *then the* more gradual *the y value* decreases. • The "a" represents the y-intercept.

Mathematical Reflection

Arc of Learning™: Synthesis, Exploration, Analysis

Standards: See your state alignment chart.

Mathematical Reflection	**Emerging Mathematical Ideas**
What do you know about exponential functions and exponential expressions? How do exponential functions compare to linear functions?	Consolidate and refine key understanding about when situations in context, tables, graphs, and equations represent exponential growth or exponential decay situations.

	Articulate how exponential decay functions are different from exponential growth functions. In exponential growth functions the numbers get increasing larger. The change happens slowly at first and then increasing more and more. In exponential decay functions the numbers get smaller The change drops quickly at first and then less and less. Articulate how exponential functions are different from linear functions, how their rate of change is multiplicative, and how dependent variables decrease by multiplication of a fixed number as the independent variable increases or decreases. Notice for growth and decay situations, entries in a table or graph are changing not by an additive pattern but rather by a decreasing multiplicative pattern. Identify the growth or decay factor and initial value in exponential situations, and be able to represent exponential situations using the form $y = a(b^x)$, where a is the initial value and b is the decay factor. Describe patterns with exponential functions: • If $b > 1$, then the graph is an increasing curve. The greater the value of b then the faster the increase is in the *y value*. • If $0 < b < 1$, then the graph is a decreasing curve. The greater the value of b *then the* more gradual *the y value* decreases. • The "a" represents the y-intercept. Recognize that expressions with exponents represent multiplicative patterns where you have a repeated factor.

Investigation 4: Patterns with Exponents

Goals

Students will be working to develop elements of the following unit goals throughout Investigation 4.

Exponential Functions (Nonlinear Functions). Explore problem situations in which two or more variables have an exponential relationship to each other.

- Identify, compare, and solve problem situations that can be modeled with an exponential function (nonlinear) or linear functions using a table, graph, or equation.
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation and make connections among the representations and to representations of linear functions.
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function

Properties of Exponents. Develop understanding of equivalent exponential expressions.

- Develop properties for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation that represent the dependent variable in an exponential function.
- Write and interpret equivalent expressions using the properties for exponents and operations
- Recognize the role and use properties of exponents and scientific notation to solve problems that involve exponential functions

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<table>
<tr><td colspan="2">Problem 4.1. Looking for Patterns Among Exponents
Arc of Learning™: Synthesis, Exploration, Analysis, Synthesis
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
What patterns did you observe in the table of powers?</td><td>Emerging Mathematical Ideas
Examine patterns involving numerical expressing containing exponents, and look for relationships among numbers written in exponential form.
Begin to notice when operating on numerical expressions with exponents, for any value of a
• $a^m \cdot a^n = a^{m+n}$;
• $a^0 = 1$; and
• $a^{-1} = \frac{1}{a}$.</td></tr>
</table>

<table>
<tr><td colspan="2">Problem 4.2. More Patterns: Properties of Exponents
Arc of Learning™: Synthesis, Analysis, Synthesis
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
Describe some properties for working with exponents. Explain why they work.</td><td>Emerging Mathematical Ideas
Recognize and use patterns for numbers written in exponential form. Begin to articulate understanding of properties of exponents, such as for any value of a and b:
• $a^m \cdot a^n = a^{m+n}$;
• $\frac{a^m}{a^n} = a^{m-n}$;
• $a^0 = 1$;
• $a^m \cdot b^m = ab^m$;
• $(a^m)^n = a^{mn}$; and
• $a^{-m} = \frac{1}{a^m}$.</td></tr>
<tr><td colspan="2">Problem 4.3. Water Usage: Operations with Scientific Notation
Arc of Learning™: Synthesis. Analysis, Synthesis
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How do the properties of exponents help you work with scientific notation?</td><td>Emerging Mathematical Ideas
Use properties of exponents to write and interpret equivalent expressions, including some in scientific notation.
Begin to use properties of exponents to solve problems with numbers written in scientific notation.
Connect to earlier work on ratios and simplifying ratios (Comparing and Scaling).</td></tr>
<tr><td colspan="2">Problem 4.4. Growing Amoebas: Extending the Properties of Exponents
Arc of Learning™: Synthesis, Analysis, Synthesis
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How does scientific notation help to solve problems?</td><td>Emerging Mathematical Ideas
Introduce rational exponents and nth roots numerical expressions to interpret and evaluate expressions. Begin to think about and use properties of integer exponents to evaluate these expressions.</td></tr>
<tr><td colspan="2">Mathematical Reflection
Arc of Learning™: Synthesis, Analysis, Synthesis
Standards: See your state alignment chart.</td></tr>
<tr><td>Mathematical Reflection
What do you know about exponential functions and exponential expressions? How do exponential functions compare to linear functions?</td><td>Emerging Mathematical Ideas
Consolidate ideas about how and why exponential function are nonlinear: linear functions have an additive pattern, and exponential functions have a multiplicative pattern.
• Note that numerical expressions containing exponents represent a multiplicative pattern where you have a repeated factor.
Properties for rational exponents are the same as those for integral exponents. The properties help you to simplify expressions with exponents.</td></tr>
</table>