

Over the past three to four decades, a growing body of knowledge from the cognitive sciences has supported the notion that students develop their own understanding from their experiences with mathematics. The National Research Council, among other groups, has drawn attention to research that suggests that “learning is a complex cognitive process that builds on prior knowledge and requires active engagement with new situations” (National Research Council, 2000). “The process of inquiry, not merely giving instruction, is the very heart of what teachers do.”

Rationale for a Problem-Centered Curriculum

- **CMP is problem-centered.** This means that important mathematical ideas are embedded in engaging problems. Students develop understanding and skill as they explore a coherent set of problems, individually, in a group, or with the class. “Effective instruction models good thinking, provides hints, and prompts students who can not get it on their own”. Inquiry, reflection, meaningful problems in a variety of contexts, and sense making are all elements of the CMP program.
- **Students' perceptions** about a discipline come from the tasks or problems in which they are asked to engage. For example, if students in a geometry course are asked to memorize definitions, they think geometry is about memorizing definitions. If students spend a majority of their mathematics time practicing paper-and-pencil computations, they come to believe that mathematics is about calculating answers to arithmetic problems as quickly as possible. They may become adept at quickly performing specific types of computations, but they may not be able to apply these skills to other situations or to recognize problems that call for these skills. If the purpose of studying mathematics is to be able to solve a variety of problems, then students need to spend significant time solving problems that require thinking, planning, reasoning, computing, and evaluating.
- **CMP places important mathematics in problems in context.** Research evidence from the cognitive sciences supports the theory that students can make sense of mathematics if the concepts are embedded within a context or problem. If time is spent exploring interesting mathematical situations, reflecting on solution methods, comparing methods, and examining why methods work, then students are likely to build more robust understanding of mathematical concepts and procedures.
- A **problem-centered curriculum** not only helps students make sense of the mathematics but also appears to help them process the mathematics in a retrievable way.
- Teachers of CMP report that **students in succeeding grades remember and refer to a concept, technique, or strategy by the name of the problem** in which they encountered the idea.
- Results from the cognitive sciences also suggest that **learning is enhanced if it is connected to prior knowledge** and is more likely to be retained and applied appropriately to future learning.
- **CMP units build on each other.** Concepts developed in one unit are deliberately connected to prior investigations and skills, and problems in future units further develop or refine strategies.

The Family Role

As families talk to their students about what they have learned in class, they become an active part of the learning process. They are some of the knowledgeable experts in their student's environment. Their expertise may be in the mathematical ideas or in the learning process itself. They can provide the help their students need with the homework, without taking away the gains to be made from a student's individual work. They can encourage their students to reflect on what was recently learned. When they ask questions and allow their students to explain concepts, they are part of the metacognitive process (reflecting on one's understanding and thinking) that researchers tell us enhances achievement and develops the ability to learn independently.

Summary

In CMP, important mathematical ideas are identified. Each idea is studied in depth within a unit and then used throughout the remaining units. These mathematical ideas are embedded in the context of interesting problems. As students explore a series of connected problems, they develop understanding of the embedded ideas and, with the aid of the teacher, abstract powerful mathematical ideas and problem-solving strategies. CMP students are developing mathematical habits of mind: solving problems, reflecting on solution methods, examining why the methods work, comparing methods, generalizing methods, and relating methods to those used in previous situations. Every problem in *Connected Mathematics* satisfies all of the following criteria:

- It contains important, useful mathematics.
- It requires higher-level thinking and problem solving.
- It contributes to students' conceptual development.
- It connects to other important mathematical ideas.
- It promotes the skillful use of mathematics.
- It can be approached in multiple ways, using different solution strategies.
- It provides an opportunity to practice important skills.
- It engages students and encourages discourse.
- It has various solution methods or allows different decisions or positions to be taken and defended.
- It creates an opportunity for the teacher to assess what students are learning and where they may be experiencing difficulty.

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