PREPARING TO IMPLEMENT CMP

This section will help to plan for the school year from unit to unit and problem to problem. Planning to facilitate lessons using the Launch—Explore—Summarize (LES) instructional model will be discussed in detail along with ACE, the Mathematical Reflection, pedagogical strategies, key terms, and differentiated instruction.

The following documents are valuable resources for planning and teaching CMP4 and can be found at the end of this part:

Guide Aid 3.A: Pacing Overview by Grade

Guide Aid 3.B: Grade 6 Goals, Now What Do You Know?, and Mathematical Reflection

Guide Aid 3.C: Grade 7 Goals, Now What Do You Know?, and Mathematical Reflection

Guide Aid 3.D: Grade 8 Goals, Now What Do You Know?, and Mathematical Reflection

Guide Aid 3.E: Grade 6 Pedagogical Strategy Suggestions

Guide Aid 3.F: Grade 7 Pedagogical Strategy Suggestions

Guide Aid 3.G: Grade 8 and Algebra Pedagogical Strategy Suggestions

Guide Aid 3.H: CMP Instructional Model Overview

Guide Aid 3.I: Questions Students Can Ask in the Explore

Guide Aid 3.J: Examples of Grading from CMP Teachers

Guide Aid 3.K: Example of Mathematical Reflections Responses

Guide Aid 3.L: Grade 8 and Algebra I/High School Course 1 Pathways

Videos from classrooms, with teachers talking with teachers, and with a teacher planning a unit are available on the Connected Mathematics website (https://connectedmath.msu.edu/).

Planning the Year

It may be helpful to develop a timeline with beginning and ending dates for each unit, including time for assessments, as well as for local and state standardized testing. The Teacher Edition provides a suggested number of class periods for each unit's completion. Guide Aid 3.A: Pacing Overview by Grade can assist in mapping out the year. As the year progresses, teachers may need to refine the timeline.

Planning a Unit

The first stage in planning to teach a unit is becoming familiar with the key concepts and the way the unit develops concepts, reasoning, and skills. In general, the unit subtitle gives a broad view of the important ideas that will be developed. For example, the grade 7 unit *Moving Straight Ahead* has the subtitle *Linear Relationships, Expressions, and Equations*, which identifies linear relationships as the central idea. What the title does not reveal is what aspects of linear relationships are developed and how understanding is enhanced. The following suggestions can serve as a guide for getting to know a unit at this more detailed level.

- Understand the mathematics and how it is being developed.
  - Read the Goals of the unit (Guide Aids 3.B, 3.C, and 3.D have the Goals, Mathematical Reflections, and Now What Do You Know? for each unit), Unit Description and the Mathematical Overview.
  - The Unit Alignment chart has Emerging Mathematical Ideas listed for each problem. These can help in knowing where students are in their understanding of each Now What Do You Know? and the Mathematical Reflection. They can also be helpful in planning for differentiation in the classroom.

- Read the Summary of Investigations and the Mathematical Reflection. These outline the development of the mathematics in the unit.
- Examine the *Arc of Learning™* for the unit. It will highlight and guide the development of mathematical understanding of each unit.
- Look over the Assessment Resources. They provide an overview of what students are expected to know at various points in the unit and show the level and type of understanding students are expected to develop.
- Work all the problems and ACE for each investigation.
- Make use of the help provided in the Teacher Edition.
  - The At a Glance provides an overview of the lesson that includes a description of the context of the problem, the materials needed, the Arc of Learning phase(s), and the Now What Do You Know? question.
  - The Extended Launch—Explore—Summary provides support and suggested questions for each phase of the instructional model.
  - The answers embedded in the Student Edition offer insights into how students will think about solving the problems.
  - Learning Aids and Teaching Aids provide teachers and students support while working through a problem.
  - A correlation of standards is provided.
- The STEM problem format guides the mathematical development of each problem. It promotes student engagement and learning as students collaborate to design solutions, make conjectures, offer critiques, and communicate their mathematical understandings. The STEM problem format provides teachers with flexibility to carry out equitable practices that help address the individual needs of all students.
- Use the Launch—Explore—Summarize instructional model as a guide for teaching each problem.
- Keep notes on important ideas or suggestions for the next time you teach the unit.
- Essential key terms are developed during each unit. Plan for time for students to record their definitions with specific examples for the terms as they occur in the unit. Displaying each key term with student displays of learning on the memory wall (described in Preparing Your Classroom Environment) will enable students to have a readily accessible reference when engaged in discourse around problems.
- Use the Mathematical Reflection as a benchmark for your students' understanding.
- Reevaluate where your students are each day. Teacher reflections are an important part in becoming a more effective teacher.

The following questions can guide the planning of a unit.

- What are the big mathematical ideas of this unit?
- What do I want students to know when this unit is completed?
- What mathematical key terms does this unit bring out?
- What might be conceptually difficult?
- What are important connections to other units?
- How might one unit give leverage for the next?

Planning an Investigation

When planning an investigation, it is helpful to read the description of each investigation, the essential understandings of the mathematical goals that are being addressed in the investigation, the Now What Do You Know? questions, and the Mathematical Reflection.

The following questions can assist in uncovering the developmental and learning trajectories in each investigation.

- What part of the main mathematical goal of the unit is being developed in the investigation?
- How does each problem in the investigation contribute to the development of the mathematics?
- What level of sophistication do I expect all my students to achieve in answering the questions in the investigation?
- What opportunities exist in the investigation to extend and connect the students' learning beyond the basic expectations?
- What type of student responses will show development in understanding the big ideas of the unit?
- What mathematical ideas will need emphasis?
- What connections can be made among the problems in this investigation, to other investigations in this unit, and to other units?
- How can I prepare for and structure the Mathematical Reflection to maximize learning?
- What ACE are appropriate for my students to do, and when will they best fit?
- How long should this investigation take?
- What can I do to ensure the amount of time spent in class is appropriate for the problems and the goals of the investigation?

At the end of each investigation, a whole-class conversation around the Mathematical Reflection is important. Students should have the opportunity to discuss their understanding and then record their individual responses to the questions in their notebooks or journals. Teachers can also use an anchor chart to have a running record of the class learning around the Mathematical Reflection as the unit progresses. The guiding question in the Mathematical Reflection provides students an opportunity to record what they know as they move through the unit.

Planning a Lesson

Begin by actually doing the problem yourself. The learning process that our students experience will become evident and you will be able to anticipate where students may struggle. It is important to know how the problem connects to prior learning as well as how it leads into the next problem. The teacher's role during a typical daily lesson is to facilitate student learning and to orchestrate the lesson. Within each phase of the lesson, the teacher listens and poses questions to elicit student reasoning.

The students' role during a typical daily lesson is to explore a concept or skill with a problem-solving mindset. Within each phase of the lesson, students listen to each other's ideas and pose clarifying questions to each other. This exchange deepens their understanding of a concept or skill. Students actively share their problem-solving strategies and conclusions. Recording their reasoning and conclusions is a key element of students being able to share the ownership of their learning with others. Creating a visual display of learning will occur regularly in each investigation.

Important resources:

- The Attending to Individual Learning Needs framework (See Part 4, A Curriculum for All Students). The specific strategies in the framework are included in the At a Glance and Extended Launch—Explore—Summarize for each problem in the Teacher Edition.
- The Pedagogical Strategies (See Part 2, What's in CMP, Guide Aid 2.K) are important resources to ensure productive engagement. Suggestions for using these strategies are included in the At a Glance and Extended Launch—Explore—Summarize for each problem in the Teacher Edition and in Guides 3.E, 3.F and 3.G.

Launch The Launch ensures that all students have access to the context and content to explore that lesson's big idea, concept, or procedure. Connecting to prior knowledge and encouraging them to delve into the task without giving away too much of the problem is the challenge. A new component of CMP4 is the Now What Do You Know? question. Some teachers have posed the question to the whole class during the Launch as a challenge or advance organizer or during the Explore as a way of deciding if it is time to summarize. Students should be able to respond to the question during the Summarize. The challenge is to pique students' interest in the problem to ensure a lively, productive exploration.

When planning the Launch, the following questions are helpful:

- What prior knowledge do my students need to call upon?
- How will I launch this problem to ensure that all students have access to begin the task?
- Do I need a mini-summary after the Initial Challenge before the What If . . . ? situations?
- What do the students need to know to understand the story and the challenge of the problem?
- What advantages or difficulties can I foresee?
- How can I keep from giving away too much of the problem?
- How can I make it personal to them?

Often a Launch can begin with asking students what do they notice and what do they wonder about the introduction or when first looking at the Initial Challenge of a problem. Many problems can also begin by having students make a prediction on what will happen in the problem.

Explore The purpose of the Explore phase of the lesson is for students to explore a rich problem, which will enable them to analyze and generalize a concept or skill. Students may tackle the mathematical task individually, with a partner, or with a small group, depending on the challenge and format of the problem. The students' role during the Explore phase is to delve into the problem. When appropriate, students should collaborate with their peers to make sense of what the questions are asking and to make a visual display of their learning for others to consider during the Summarize.

The teacher's role during this phase is to provide for individual needs and to plan for the Summarize phase. To do this, the teacher must observe and listen to the students at work. The teacher will pose questions to support struggling students, taking care to follow the students' thinking rather than the teacher's pathway of reasoning. The teacher will pose questions to push the thinking of those students who quickly complete the task and demonstrate that they are ready to be challenged further. Consideration must be given to when to end the Explore. Also, the

teacher needs to decide the opening question for the Summarize. A teacher must plan which students' reasoning and conclusions, and which sequence, will provide a stimulating discourse on the mathematics.

See Part 2, What's in CMP, Guide Aid 2.L: *The Formative Assessment Framework* for more suggestions.

When planning the Explore, the following questions are helpful:

- Which parts of the What If . . . ? should be assigned to all students? How can I use this part to provide differentiation?
- Will I organize the students to explore this problem individually, as pairs, or as small groups? Or can they organize themselves to best engage the task?
- What materials should be made available for students?
- Will students display their learning in individual papers, on chart paper, on construction paper, or by oral presentation?
- What are different strategies I anticipate students using?
- What kinds of questions can I pose to prompt their thinking if the level of frustration is high?
- What kinds of questions can I pose to make them probe further into the problem?
- What kinds of questions can I pose to encourage student-to-student conversation about their thinking, reasoning, and learning?
- What kinds of strategies will I be looking for to facilitate the summary, and how will I sequence them to stimulate a productive summary?
- How will the Now What Do You Know? help to decide when it is time to end the Explore?

Summarize The purpose of the Summarize phase of the lesson is to orchestrate whole-group student discourse about students' discoveries during the Explore phase of the lesson. It is during the Summarize that the teacher guides the students to reach the mathematical goals of the problem and to connect their new understanding to prior mathematical goals and problems in the unit. Student conjectures and conclusions are shared and considered by their peers. Conclusions are solidified, and sometimes questions are posed for further exploration in future lessons. The teacher begins by imposing an opening question that will get the conversation started. After that, the students should lead the Summarize by presenting their conjectures and conclusions with mathematically sound support. The teacher and students ask clarifying questions. Students draw conclusions addressing the mathematical goals of the lesson.

When planning the Summarize, ask yourself the following:

- How can I help the students make sense of and appreciate the variety of methods that may occur?
- How can I orchestrate the discussion so students lead the summary of their thinking in the problem?

- What mathematics and processes need to be drawn out in discussing the Now What Do You Know? or the Mathematical Reflection?
- How will these ideas be recorded?
- Which ideas need to be made visible and displayed for future reference?
- What needs to be emphasized?
- What ideas do not need closure at this time?
- What do we need to generalize?
- How can we go beyond? What new questions might arise?
- What will I do to follow up, practice, or apply the ideas after the Summarize?

Reflect At the end of each lesson, it is productive to take note of student understandings:

- What evidence do I have of what my students learned about the Now What Do You Know? question?
- What revelations occurred?
- What struggles did students have?
- How does this affect my instructional decisions for the next lesson? For the next time I teach this problem?
- Where will I have time to revisit these ideas in the next problem, investigation, or unit?

See Part 3, Planning and Teaching, Guide Aid 3.H for a summary of the Launch–Explore–Summarize.

	CMP Instructional Model		
	Launch	Explore	Summarize
Teacher Role	• Set up the challenge • Help position the problem within prior understandings and experiences	• Move about the classroom, observing and interacting with individual and small groups. • Ask appropriate questions, providing students with confirmation and redirection where needed.	• Discuss the solutions and strategies • Make the mathematics explicit • Connect their new understandings to prior mathematical goals
Student Role	• Introduced to the context and challenge of the problem during whole-class discussion. • Make predictions and ask clarifying questions about the problem situation.	• Work collaboratively to explore and solve the problem in small groups. • Gather data, share ideas, look for patterns, make conjectures, develop strategies and create arguments to support their reasoning and solutions	• Present and discuss their solutions and strategies during a whole-class discussion • Discuss the embedded or encoded mathematics of the problem • Connect learning to prior and future knowledge

Did You Know?

Many curriculum materials in mathematics education use or adapt the Launch–Explore–Summarize instructional model. In 1980, the National Science Foundation (NSF) funded the Middle Grades Mathematics Project (MGMP). MGMP is a curriculum project developed at Michigan State University. MGMP consists of five middle school mathematics units, the *Mouse and Elephant, Factors and Multiplies, Similarity, Spatial Visualization, and Probability* which are still in print. These units formed the basis for the Connected Mathematics Project (CMP) middle school mathematics curriculum which was also funded by NSF. Both MGMP and CMP are problem-based curriculum. To help the teacher implement the curriculum requires extensive teacher support. One of the supports that was developed by the MGMP authors is an instructional model that consists of three phases: Launch–Explore–Summarize.

Assigning Applications–Connections–Extensions (ACE)

The Applications–Connections–Extensions (ACE) are written to provide additional learning opportunities for your students. A rule of thumb is to answer the ACE yourself with the level of detail that you expect from your students while timing yourself. Take that time and multiply it by three or four to estimate how long it will take your students to complete. There are suggestions in the At a Glance pages in the Teacher Edition for each unit for which ACE align with each of the investigation's problems.

When assigning ACE for homework, ask yourself the following:

- Why homework?
- What do I intend for my students to gain from the homework assignment?
- How will it further our collective learning?
- How will students get feedback about their work on the ACE?

Consider how much time you will devote to homework correction both during class and outside of class and the time you will need to spend planning and implementing the daily lessons. You may also consider offering different ACE assignments to different students that align with where they are in their learning.

- Often teachers select an ACE to use as an entry activity or exit slip. One procedure is to have students respond to the question on mini-whiteboards they hold up for a quick assessment. Another procedure is to have students respond on a sticky note and stick it on the door as they exit class. You can readily sort their responses to assess who is ready to build on the concept or skill and who may need some additional practice or support.
- Teachers can select various ACE as additional learning opportunities for students. These exercises may be chosen by the teacher or selected by individual students. To help students choose, you may point out that the ACE are designed to help students become more confident in their understandings. Students who believe that they would benefit from additional practice choose from the exercises that align with the day's lesson. Teachers can also use the ACE to differentiate instruction for various learners. Also different ACE can be assigned to individual students who need additional practice to solidify their understandings or need an additional challenge to deepen their understanding beyond the class discussion.

GUIDE AID 2.K

CMP4 General Pedagogical Strategies[1]

CMP classrooms represent a wide range of diversity in terms of gender, race, ethnicity, economic status, or disability. Evidence suggests that CMP can be and has been successfully implemented in classrooms that include students with special needs, academically gifted students, historically and culturally underrepresented students, and multilingual students. Below are a list and descriptions of the pedagogical strategies that CMP teachers have used effectively in CMP classrooms. They promote deep productive engagement around the important mathematical concepts and reasoning. While the strategies provided are general suggestions for CMP classrooms, many of these strategies have been found effective for specific populations of students. For more information on strategies for students with special needs, academically gifted students, historically and culturally underrepresented students, and multilingual students, please see Part 4, A Curriculum for All Students.

The general pedagogical strategies are found in the CMP4 Teacher Edition, throughout the Extended Launch—Explore—Summarize and the At a Glance. For each problem, there are suggestions that refer to the teaching strategies found in the list.

- Anchor Charts
- Agree/Disagree
- Chalk Talk
- Claim, Support, Question
- Collaborative Chart Paper
- Compare Thinking
- Connect, Extend, Challenge
- Delay the Question
- Fist-to-Five
- Gallery Walk
- Going Vertical
- Independent Think Time
- I Used to Think . . . Now, I Think . . .
- Make Predictions
- Non-Permanent Work Surfaces
- Notice Wonder
- Restating
- Rough Draft Mathematics
- Selecting and Sequencing
- Sentence and Question Stems
- Stop and Jot
- Think, Pair, Share
- Three Reads
- Turn and Talk
- Wait Time
- References

Anchor Charts

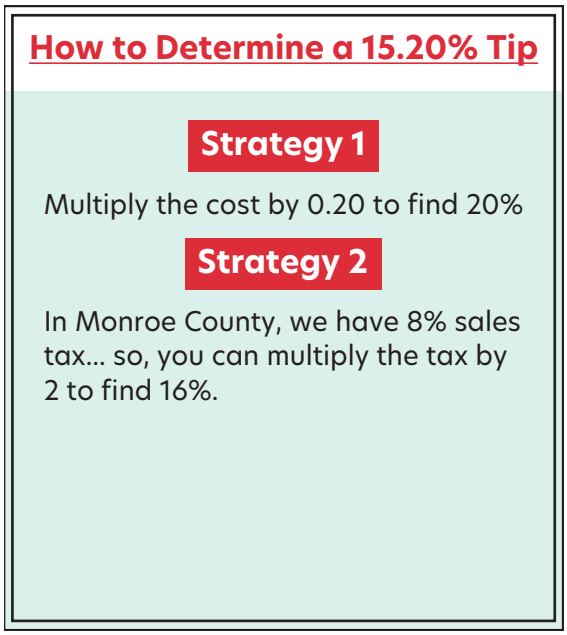
Creating Anchor Charts to display in the CMP classroom aids in making thinking visible to all students by recording strategies and essential content from class discussions. With student input, a teacher records important information from discussions using student voice and language. The teacher then posts the Anchor Charts in a visible place in the classroom that can be accessible to students throughout a unit. By posting important information from class discussions, students can access prior learning and make connections to new learning

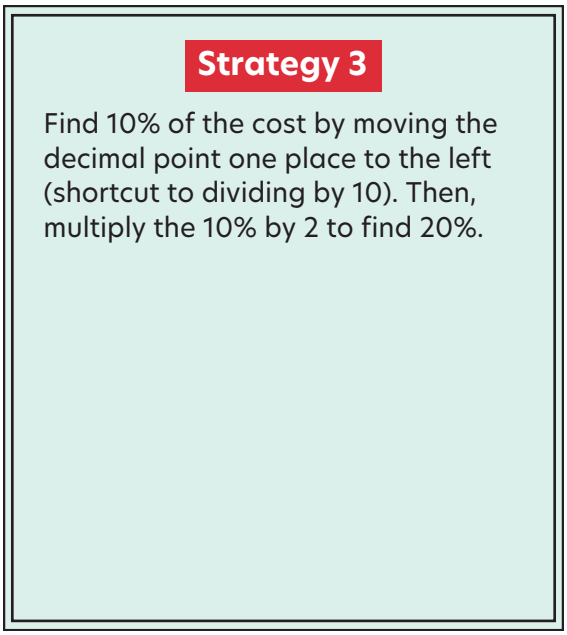
[1] The CMP Development Team wishes to acknowledge and thank Jennifer Krueger's substantial work on this document.

(EL Education, 2014). Anchor Charts should be considered a work in progress. Therefore, as new information is presented to students, Anchor Charts should be updated and revised to show new or expanded understanding. Many CMP teachers use Anchor Charts throughout an entire unit and keep them posted for assessments so students can use them as visual cues to apply problem-solving strategies.

Below are examples of Anchor Charts created by a class during the Launch of Problem 3.2 of the grade 7 unit *Comparing and Scaling*. Before launching the problem, students asked teenagers or adults how they determined tips in a restaurant without using a calculator. The teacher recorded the ideas the students brought back to the class on these Anchor Charts, which were then posted in the classroom for the remainder of the unit.

Launch of Problem 3.2 of the grade 7 unit *Comparing and Scaling*





Agree/Disagree

Agree/Disagree is a teacher move that encourages students to interact with their classmates' reasoning, strategies, and claims. After a student shares an idea, strategy, or claim, the teacher asks the class if they agree or disagree with what has been shared and asks them to explain why (Saphier & Haley-Speca, 2015). Some possible prompts a teacher might use include the following:

- Do you agree or disagree with Michael's claim? Why?
- Please give a thumbs-up if you agree, a thumbs-down if you disagree, or a thumb in the middle if you are unsure.
- Who agrees with this strategy? Who disagrees? Who would like to defend their choice?

Additionally, a teacher might ask if a student agrees with, disagrees, with or wants to add on to a claim, strategy, or idea. By asking students to add on, the teacher encourages students to engage more fully in the discussion by interacting with others' ideas and how they do or do not connect to one's own way of thinking (Chapin et al., 2013).

For example, in Problem 1.1 of the grade 8 unit *Looking for Pythagoras*, a student might make the following claim about driving distance compared to flying distance:

> *"The distances are similar when you go in a straight line, but a helicopter can cut corners and does not have to be on the street like a car does. So a helicopter can get to the same place in a shorter distance."*

After the student shares their response above, the teacher might ask the class, "Do you agree or disagree, or would you like to add on to this claim?"

When a teacher uses these moves in their instruction, they need to consider how they can support the development of skills students will need to respectfully express agreement and disagreement with their peers and listen in ways that will allow for meaningful add-ons. Setting classroom norms for conversations can help elicit more meaningful conversations.

Chalk Talk

A *Chalk Talk* can be implemented during any phase of a CMP lesson but is likely most effective during the Summarize phase. A Chalk Talk is a silent activity that allows students to reflect, think, and generate ideas around given prompts (Ritchhart et al., 2011).

The teacher begins a Chalk Talk by writing prompts on several pieces of chart paper and displaying the chart paper around the classroom (Ritchhart et al., 2011). Students are given markers and asked to remain silent for the duration of the activity. Students walk around the classroom and visit posters. They may write an idea related to the given prompt on each poster, or they may respond to a classmate's comment that is on the poster. Students can respond to a classmate's comment by writing a comment next to it, drawing a line to connect similar ideas, or drawing emojis or pictures to represent their thoughts or feelings (NRSF, 2014). Students may not return to their seats until the activity is over. Students may either be writing on the posters or reading and reflecting on the comments for the duration of the activity.

Throughout the duration of a Chalk Talk, the teacher should remind students to stay silent and active. Additionally, a teacher might write their own comments or questions on the posters to challenge students' thinking or push students to think in different ways (NRSF, 2014).

For example, a teacher might use a Chalk Talk at the end of Investigation 2 of the grade 7 unit *Completely Rational* to assess students' understanding of adding and subtracting rational numbers by placing the following statements on chart paper and asking students to respond to the statements and decide whether they are true or false:

- A positive number plus a negative number is always negative.
- A negative number minus a negative number is sometimes negative.
- A positive number minus a negative number is sometimes negative.
- A negative number plus a negative number is always negative.

Claim, Support, Question

Claim, Support, Question is a strategy that supports students in all phases of a CMP lesson and can be used individually, in small groups, and in whole-group discussions. This strategy should become an integral part of the CMP classroom, as it requires students to form a claim, support the claim with evidence, and question their claim to deepen understanding (Ritchhart et al., 2011). To implement this strategy, the teacher asks students to

- make a claim or explain their strategy or solution;
- provide evidence that supports the claim; and
- ask questions related to the claim and supports.

Claim, Support, Question encourages students to thoughtfully examine statements rather than accepting them as fact (Ritchhart et al., 2011). By questioning their claims and supports, students see reasoning as an iterative process and deepen their understanding beyond the surface level.

The CMP curriculum often requires students to provide an answer and give evidence to support the answer. However, you can extend these opportunities to have students ask questions related to their claims and supports. For example, in Problem 2.1 of the grade 7 unit *Moving Straight Ahead*, students are asked the following question.

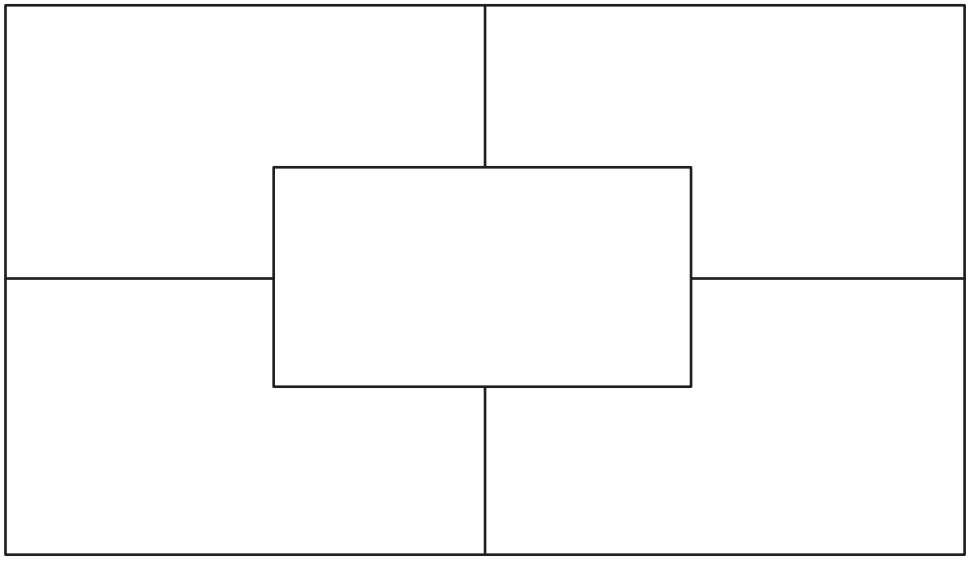
From Grade 7, Problem 2.1 in the *Moving Straight Ahead* Unit

> Henri challenges Emile to a walking race. Because Emile's walking rate is faster, Emile gives Henri a 45-meter head start. Emile knows his brother would enjoy winning the race, but he does not want to make the race so short that it is obvious his brother will win.
>
> - How long should the race be so that Henri will win in a close race? Give evidence to support your answer.

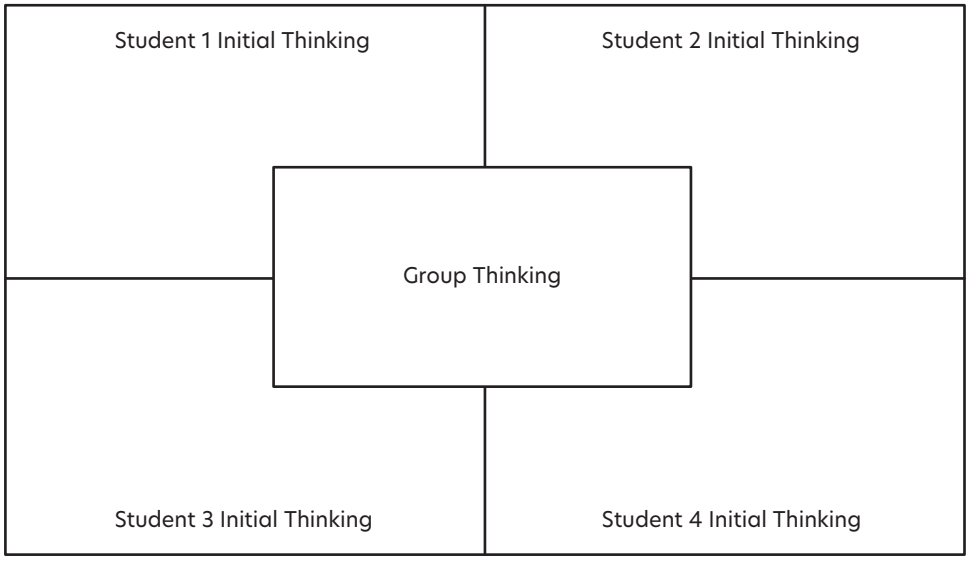
While the curriculum already provides students with the opportunity to develop an answer and provide evidence to support the answer, the teacher can add on the third part of this strategy and require students to ask questions related to their claim and supports.

Collaborative Chart Paper

When having students work in collaborative groups on a problem, giving them time and space to think about the problem independently and then having a space to record their initial thoughts is helpful. After working independently, students should share their initial thoughts and decide which strategy or strategies they would like to continue pursuing as a group. One way a teacher might have students record their initial thinking and the group's shared thinking is by using a *Collaborative Chart Paper*. To do this, a teacher would split a large sheet of chart paper in the following way:



While working during independent think time of the Explore phase, each student would write their initial thoughts in one of the outer sections of the chart paper. As students begin working in groups, each student would share their initial thinking. After the group decides which strategy they would like to continue pursuing, they would write their group thinking in the inside rectangle (see image below).



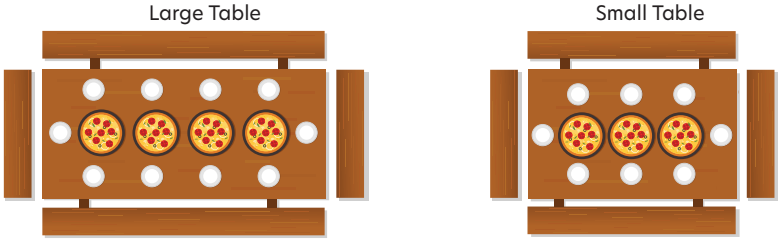
For example, in Problem 3.1 of the grade 8 unit *Mars, Gravity, and Painted Cubes*, students begin working independently to look at the pattern in the triangular numbers and then come together as a group to finalize how to describe the pattern.

Compare Thinking

Compare Thinking is a strategy that can be used during any phase of a CMP lesson but is likely most effective during a Summarize discussion. To implement this strategy, the teacher has students examine two or more ideas, strategies, or solutions (Research for Better Teaching, 2014). The teacher asks the students to comment on the similarities and differences between the different approaches or ways of thinking.

For example, the following two strategies might be shared during the Summarize discussion of Problem 2.1 of the grade 7 unit *Comparing and Scaling*. The teacher would then ask the students to examine the strategies and share similarities and differences between the two strategies.

From Grade 7, Problem 2.1 in the *Comparing and Scaling* Unit



- Does a person sitting at a small table get the same amount of pizza as a person sitting at a large table? Explain your reasoning and conclusion.

Student A

Large Table:

10 people ÷ 4 pizzas = 2.5 people

- 10 people at large table
- 4 pizzas

2.5 people × 4 pizzas = 10 people

Small Table:

8 people ÷ 3 pizzas = 2.6 people

- 8 people
- 3 pizzas

2.6 people × 3 pizzas = 8 people

2.6 people is less people you have to choice the pizza with than 2.6. So, the large table is better to sit at.

Student B

4 ÷ 10 = 0.4 Each person gets 0.4 of a pizza

3 ÷ 8 = 0.375 Each person gets 0.375 of a pizza

0.4 > 0.375
Each person gets more pizza at the large table

Connect, Extend, Challenge

Connect, Extend, Challenge can be utilized during the Summarize of a CMP lesson, at the end of an investigation, or at the end of a unit to assist students in making connections between new ideas and prior knowledge through reflective thinking. This strategy provides a structured protocol for students to reflect on new learning by articulating ideas that connect with the prior understanding of a topic, ideas that expand and lead to possible revisions in thinking about a topic, and ideas that are confusing about a topic (Ritchhart et al., 2011).

A teacher implements this strategy by asking students to respond to three questions:

- How are you connecting the new ideas to what you already know?
- How do these new ideas extend your thinking in new directions?
- What aspects of these new ideas are challenging your thinking?

Connect, Extend, Challenge can be implemented in a whole-class discussion, in small groups, or independently. The teacher might choose to keep a visible record of students' thinking and revise as students' comprehension of a concept develops over time. This strategy encourages students to reflect in new ways and question assumptions they may have made in the past. Connect, Extend, Challenge supports students in developing self-awareness of where they currently may be in a learning trajectory of a particular concept.

For example, Connect, Extend, Challenge could be implemented at the end of Investigation 1 of *Flip, Spin, Slide, and Stretch* after students have been introduced to reflections, rotations, and translations. Engaging in this strategy at the end of the investigation allows students to connect their new learning of transformations to prior knowledge from elementary school.

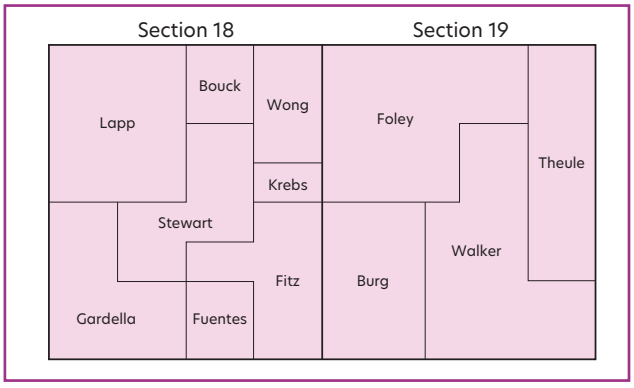
Delay the Question

Delay the Question is a strategy teachers can use that involves sharing the premise of a problem without giving a question. This strategy allows students to focus on making sense of a relationship in a problem without the distraction of a specific question being asked (Barlow et al., 2017). To initiate this strategy, a teacher shares information or an image from a problem but does not give students any mathematical question(s) to answer. Students are asked to share what relationships they notice or what they see in the information presented. This allows

teachers to ask questions that focus students' attention on specific relationships between important quantities in the problem (Barlow et al., 2017).

For example, when launching Problem 1.2 of the grade 6 unit *Bits of Rational*, the teacher might present the map of Tupelo township below and just ask students to share what they notice about how the different pieces of land relate to each other.

From Grade 6, Problem 1.2 in the *Bits of Rational* Unit



Fist-to-Five

Fist-to-Five is a strategy teachers can use in any phase of instruction during a CMP problem to provide a quick visual of student understanding, comfort, or agreement. A teacher might ask students any of the following to initiate a Fist-to-Five:

- Where is your understanding of this concept at this point in time?
- To what extent do you agree with the following statement . . .?
- How confident are you in your answer?

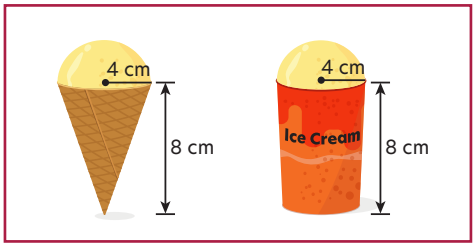
The teacher asks students to hold a fist to their chest so their answer is not public for all to see. After asking a question, such as one of the above questions, students hold up a fist for disagreement, low confidence, or low understanding or one to five fingers for higher levels of confidence, understanding, or agreement (EL Education, 2014).

For example, a teacher might use a Fist-to-Five in the following ways during a CMP lesson:

- At the end of the Launch, a teacher might ask students to express their level of readiness to start exploring the problem.
- During the Explore, the teacher might ask students to express their level of readiness to share strategies or their level of needed assistance from the teacher.
- At the end of the Summarize, a teacher might ask students to express their level of understanding.

For example, when getting ready to discuss the volumes of cones, spheres, and cylinders during the Summarize of Problem 2.4 in the grade 8 unit *Say It with Symbols* a teacher could use Fist-to-Five to ask students to express their level of confidence in their answers.

From Grade 8, Problem 2.4 in the *Say It with Symbols* Unit



Gallery Walk

A *Gallery Walk* is a strategy that can be used during the Summarize of a CMP lesson to allow students to share their thinking with their classmates in a gallery setting (EL Education, 2014). During the Explore phase of a lesson, the teacher has groups of students display their thinking on chart paper so others can understand their reasoning. The teacher then displays the chart papers (or a selection of the chart papers) around the classroom. Students then move from poster to poster to examine the reasoning of their classmates. The teacher might provide students with sticky notes to leave a comment, a question, or a connection for the group that created the poster. Additionally, the teacher might provide a question or a reflective prompt for students to consider and take notes on as they travel from poster to poster.

For example, during the Summarize of Problem 1.4 from the grade 6 unit *Variables and Patterns*, the teacher could display the tables and graphs generated by each group. As students visit the graphs during the *Gallery Walk*, they write plusses (positive attributes), write arrows or dashes (places to improve), or ask questions to the group that created the poster.

Going Vertical

Instead of having students work at desks, the floor, the counter, or other horizontal work surfaces, *Going Vertical* has students work while standing. Students might work on chart paper hanging on the wall, whiteboards or chalkboards on the wall, rolling whiteboards, or the side of a vertical file cabinet. Liljedahl (2021) found that sitting creates a sense of anonymity for students that allows for disengagement to occur in the classroom. By having students work in vertical settings, increased engagement and collaboration occur between students, and teachers get the extra advantage of being able to review and access students' work.

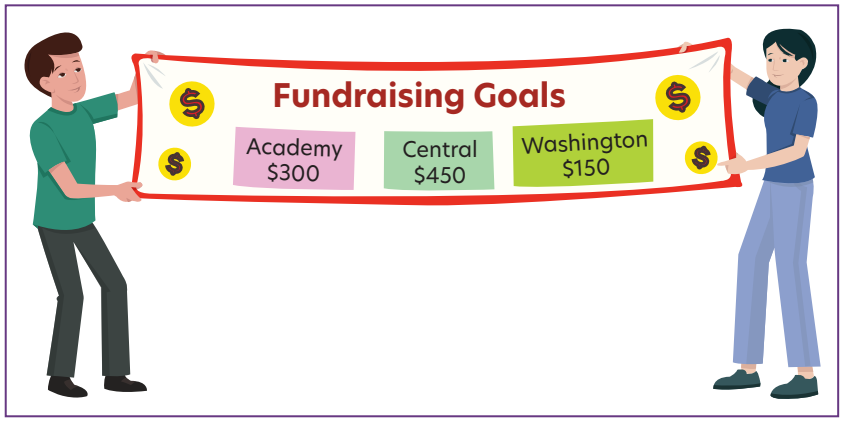
Independent Think Time

Independent Think Time (also called Private Think Time) is a time given to students to process a problem independently before working in small groups or pairs (Marchitello & Wilhelm, 2014). When teachers move directly to partner or group time before allowing students to process independently, students tend to favor the idea shared first in a group (Smith & Stein, 2018). Independent Think Time gives students time to process the information in a problem before entering a conversation with their peers.

To initiate Independent Think Time, a teacher gives students a short time (usually at most 5 minutes) to work independently on the problem. Students need to understand that they are not expected to solve the task during this short time but rather to begin considering possible pathways to a solution, begin creating a representation that might aid in solving the problem, write down important information, or write questions that they have about how to go about solving the problem (Smith & Stein, 2018). During Independent Think Time, teachers should refrain from assisting students or answering questions they have. By letting all students work independently, the teacher can provide a quiet atmosphere without interruptions.

For example, in Problem 1.1 of the grade 6 unit *Comparing Quantities*, the Initial Challenge has several student claims for students to decide if they are true. Giving students independent think time to think about the claims before they talk with their partner can assist students in having a richer conversation.

From Grade 6, Problem 1.1 in the *Comparing Quantities* Unit



I Used to Think . . . Now I Think . . .

This routine can be used during the Summarize phase of a problem or throughout a unit of study for students to reflect on how their thinking has changed and why it has changed. The teacher initiates this routine by explaining to students that they will be asked to reflect on how their thinking has changed about a specific topic over time (Ritchhart et al., 2011). The teacher then asks students to respond to the following sentence stems:

- I used to think . . .
- Now, I think . . .

By using tentative language with the word *think*, students are more likely to take risks and share their thinking, as it does not feel definitive. To expand on this routine, you might add an additional sentence stem:

- I still wonder . . .

Adding this additional prompt gives space for students to share what they are still confused about or what they still need to explore to gain a deeper understanding.

For example, in Problem 3.2 of the grade 7 unit *Stretching and Shrinking*, asking students to reflect on how their thinking of the term *mathematically similar* has changed over time by having them use the sentence stems "I used to think mathematically similar was . . ." and "Now, I think mathematically similar is . . ." gives students time to think back over their work in this unit and how their thinking has evolved.

Make Predictions

Asking students to *Make Predictions* is a strategy that can be used in any phase of a CMP lesson. When used in the Launch of a lesson, it can stimulate student interest and engagement while possibly connecting to prior knowledge. If employed during the Explore phase, we can motivate students to persevere in their exploration and problem-solving process. In the Summarize, we can return to predictions made in other phases of the lesson to compare our initial thoughts to what we have learned through exploration of the mathematics.

No matter what part of the lesson students are asked to make a prediction in, it is essential that students understand that predictions should be based on reasoning with a proper rationale and not simply a random guess (Kim & Kasmer, 2007). Additionally, a teacher should give time and space to thoroughly examine initial predictions and the reasoning behind them and time to revisit and reflect on initial predictions throughout the lesson (Kim & Kasmer, 2007).

Examples of times a teacher might use predictions in a CMP lesson are as follows:

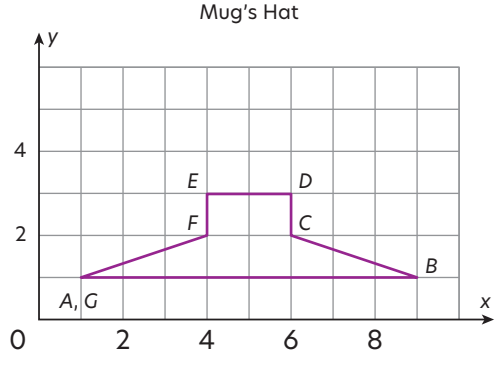
- During the Launch of Problem 4.4 of the grade 6 unit *Comparing Quantities*, teachers can ask students to Make Predictions of what grade they think the student will have with the following scores on assignments:

From Grade 6, Problem 4.4 in the *Comparing Quantities* Unit

Science Assignment	Points Correct	Points Possible
Homework 1	1	2
Checkup 1	4	5
Partner Quiz	9	10
Homework 2	4	5
Homework 3	2	3
Test	24	25

- During the Explore of Problem 1.1 of the grade unit, *Thinking with Mathematical Models,* a teacher might stop students as they are engaging in the Bridge Experiments to Make Predictions at various points throughout the experiment by continuing to ask the students, *"How do you think the strength of a beam is related to its thickness?"*
- During the Summary of Problem 2.2 of the grade 7 unit *Stretching and Shrinking*, it is important to return to the student predictions made during the Initial Challenge of what each of the following rules will do to Mug's hat compared to what actually happened with testing of each rule.

From Grade 7, Problem 2.2 in the *Stretching and Shrinking* Unit



Mug's Hat

	Mug's Hat	Hat 1	Hat 2	Hat 3	Hat 4	Hat 5
Point	(x, y)	$(x + 2, y + 3)$	$(x - 1, y + 4)$	$(x + 2, 3y)$	$(0.5x, 0.5y)$	$(2x, 3y)$

Nonpermanent Work Surfaces

Many students, especially middle and high school students, do not share their thinking in math class unless they feel they have completed thoughts that include a correct answer. Therefore, some students do not feel comfortable committing their answers to a piece of paper, a worksheet, or a large sheet of chart paper that gives the impression of permanence (Jansen, 2020). Allowing students to use dry-erase markers on non-permanent work surfaces such as whiteboards, windows, desks, or laminated chart paper allows them to easily erase incorrect

attempts, which reduces the risk of trying something new (Liljedahl, 2021). The nonpermanence of these surfaces places less emphasis on correct strategies and complete thoughts and gives students a place to explore mathematics without feeling like they are producing a final copy of their work.

For example, using nonpermanent work surfaces in Problem 3.2 of the grade 6 unit *Bits of Rational* allows students to test out their ideas on dividing mixed numbers by fractions in a setting that allows them to have errors and easily correct them.

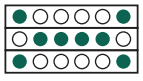
Notice Wonder

The *Notice Wonder* routine is a strategy teachers can use to give entry to all students into a problem in a low-stakes manner. To engage students in a Notice Wonder, the teacher shows an image to students with no mathematical question given. The teacher then asks students, "What do you notice?" and "What do you wonder?" Students are given the opportunity to share noticings and wonderings without the pressure of having to answer a question or employ mathematical strategies. This strategy allows all students to become engaged with the context and content of a problem before diving deeper into solving it (NCTM, 2020).

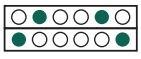
For example, to launch Problem 1.3 of the grade 6 unit *Comparing Quantities*, the teacher might show the following image and description and ask students, "What do you notice?" and "What do you wonder?"

From Grade 6, Problem 1.3 in the *Comparing Quantities* Unit

The students at Washington sell cupcakes to raise money. Since the school colors are green and white, the cupcakes are green or white. They sell them in two different-size boxes. Maria suggests using ratio statements to advertise the cupcakes.



Large Box
8 green cupcakes
10 white cupcakes



Small Box
4 green cupcakes
8 white cupcakes

Restating

Restating can be used in any phase of a CMP lesson in small groups or during whole-group discussions. To implement this strategy, a teacher asks a student to restate or paraphrase what has been said by a classmate (Research for Better Teaching, 2014). If the student is unable to restate what their classmate said, the teacher can do one of the following (West, 2016):

1. have the student ask their classmate to repeat their idea so it can be restated or paraphrased
2. tell the student they will come back to them after another classmate restates the idea (The teacher must remember to return to the original student to hold them accountable for restating the idea.)

Restating sends the message to students that all voices are important and are needed in classroom discussions. Additionally, this strategy sets a classroom expectation that students must listen to their classmates' ideas.

For example, in Problem 2.3 of the grade 8 unit *It's in the System* having students restate the strategies/solutions of other students can help move the conversations forward in creating an equivalent system in order to solve the linear system.

From Grade 8, Problem 2.3 in the *It's in the System* Unit

For each system, solve the system, and check your answer with both equations in the original system.

1. $\begin{cases} 2x + 2y = 5 \\ 3x - 6y = 12 \end{cases}$

2. $\begin{cases} x + 3y = 4 \\ 4x + 5y = 2 \end{cases}$

3. $\begin{cases} 2x + y = 5 \\ 3x - 2y = 15 \end{cases}$

4. $\begin{cases} -x + 2y = 5 \\ 5x - 10y = 11 \end{cases}$

Rough Draft Mathematics

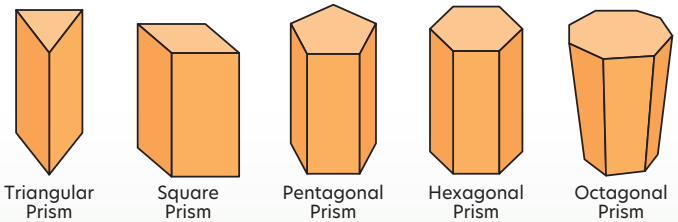
Rough Draft Mathematics should be an essential strategy for any CMP classroom. Students often believe that when they share their thinking in math class, it must be correct and complete (Jansen, 2020). This line of thinking prohibits many students from openly sharing their thinking out of fear of being incorrect or judged by classmates or the teacher. Alternatively, students thrive in a safe environment that encourages exploration and refinement of ideas. Rough Draft Mathematics is similar to rough draft thinking in English language arts and includes the following steps of drafting and revising:

- A student develops and shares initial draft ideas.
- The teacher and classmates identify strengths and areas for potential refinement in the student's draft ideas.
- The student is given the opportunity to revise their initial draft ideas.

By employing Rough Draft Mathematics in a classroom, the teacher shifts the classroom from a culture of performance to a culture of learning (Jansen, 2020). This shift allows students to make sense of mathematics in an ongoing iterative process in which students revisit and revise their initial ideas as they acquire new knowledge. It is important to note that Rough Draft Mathematics is not limited to written ideas in mathematics. This strategy can also be applied to the oral sharing of ideas.

For example, for the Now What Do You Know? in Problem 2.1 in the grade 7 unit *Filling and Wrapping*, having students share and refine their ideas on how the volume and surface area of the prism change as the number of sides increases can help them make sense of the ideas that come out of the experiment in the Initial Challenge.

From Grade 7, Problem 2.1 in the *Filling and Wrapping* Unit



Selecting and Sequencing

Selecting and Sequencing is a strategy CMP teachers can use in Summarize discussions to support the facilitation of meaningful mathematics discourse (Smith & Stein, 2018). When teachers employ this strategy, they purposefully select particular students' strategies during the Explore phase to be presented in an intentional order during the whole-class summary discussion.

Selecting student work to be shared during the Summarize discussion requires the teacher to choose strategies that are accessible and illustrate thinking toward established mathematics learning goals (Smith & Stein, 2018). When selecting student work to be shared, it is important that over time all students have opportunities to share their work in order to demonstrate competence. Teachers must recognize that not all students will share during every summary discussion. Strategies should only be selected if they build toward the mathematical learning goal of the lesson. Additionally, it is crucial that not only correct and complete strategies are shared but also misconceptions and incomplete work are shared when they contribute to student learning.

When choosing a sequence for students to present their strategies, teachers must consider how they can order the presentations of student work to build a coherent mathematics story (Smith & Stein, 2018). Teachers might consider starting with more concrete strategies and building to more abstract strategies, sharing a misconception or incomplete strategy first, or sharing the most common strategy first. When teachers Select and Sequence student work for the Summarize phase of a lesson, they establish control of the content shared and discussed.

For example, a teacher might select the following strategies from Problem 1.1 of the grade 7 unit *Comparing and Scaling* to examine during a Summarize discussion in the given sequence. The students examine four orange juice recipes and determine which one is the most "orangey" and which is the least "orangey." The sequence shown moves from a part-to-whole comparison into part-to-part comparisons that include a unit rate comparison. The sequence ends with a more unique strategy in which the student compares part-to-part ratios one at a time versus all together. ***Note: This is one possible way a teacher might choose to select and sequence strategies for this problem.***

From Grade 7, Problem 1.1 in the *Comparing and Scaling* Unit

Comparing and Scaling

Recipes	Cups of Juice	Total Cups	% of Juice
A	2	5	40%
B	5	14	36%
C	1	3	33%
D	3	8	38%

Mixture A, because it has the largest percentage of Juice.

Mix A 40% of Juice

Mixture C, because it has the smallest percentage of Juice.

Mix C 33% of Juice

ORANGE JUICE

FRACTIONS

A	B	C	D
$\frac{2}{3}$	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{3}{5}$
×30 ↓	×10 ↓	×45 ↓	×18 ↓
$\frac{60}{90}$	$\frac{50}{90}$	$\frac{40}{90}$	$\frac{64}{90}$
Most "Orangey"		Least "Orangey"	

Explain

A = A is the most "Orangey" because it has the biggest fraction, which means it has the most concentrate which will make it the most "Orangey".

C = C is the least "Orangey" because it has the least fraction, which make it better the most wattery with the least amount of "Orange".

Which juice is the most "orangey"?

A **Mix A**

$\div 2 \begin{pmatrix} 2:3 \\ 1:1.5 \end{pmatrix} \times 2$

Strongest

B **Mix B**

$\div 5 \begin{pmatrix} 5:9 \\ 1:1.8 \end{pmatrix} \times 5$

Which juice is least "orangey"?

C **Mix C**

1 : 2

Weakest

D **Mix D**

$\div 3 \begin{pmatrix} 3:5 \\ 1:1.\overline{6} \end{pmatrix} \times 3$

$A=\frac{2}{3}$ $B=\frac{5}{9}$ $C=\frac{1}{2}$ $D=\frac{3}{5}$

$\frac{2}{3}>\frac{5}{9}$, $\frac{2}{3}>\frac{1}{2}$, $\frac{2}{3}>\frac{3}{5}$

Orange

Water

Mix A is the most orangey because it is the largest fraction of concentrate to water.

$\frac{2}{3}>\frac{1}{2}$, $\frac{5}{9}>\frac{1}{2}$, $\frac{3}{5}>\frac{1}{2}$ $C=\frac{1}{2}$

Mix C is the least orangey smallest ratio/fraction of concentrate to water.

Sentence and Question Stems

Sentence and Question Stems can be utilized in CMP classrooms to support students' development of mathematics discourse. To become effective communicators of mathematics, students need help learning and utilizing academic vocabulary in ways that clearly communicate their thinking and critique others' thinking (Buffington et al., 2017). Sentence and Question Stems are partial sentences that students complete with their own ideas and words. Using Sentence Stems in your classroom allows students to respond in a complete sentence that effectively communicates thinking and reasoning. Sentence and Question Stems provide scaffolding that helps students initiate speaking without the added pressure of thinking about correctly formulating a response. It is important for teachers to explicitly model how to use Sentence and Question Stems. Sentence and Question Stems should be posted in classrooms or placed directly on students' desks in order for students to have easy access to them during small- and whole group discussions.

Example Sentence Stems to Support Student Critique:

- I agree with you because . . .
- I disagree with you because . . .
- I understand this part of your work, but I am confused by . . .
- Another way you could look at this is . . .
- Have you considered . . .
- Did you think about . . .

Example Sentence Stems to Share Thinking:

- I solved the problem by . . .
- My strategy for solving the problem is . . .
- I started by . . .
- I proved my thinking by . . .
- I know my answer is reasonable because . . .

Example Sentence Stems for Making Connections:

- I notice that . . .
- Your idea reminds me of . . .
- These strategies are similar because . . .
- These strategies are different because . . .
- I'd like to add on . . .

Example Sentence Stems to Ask Questions and to Clarify Thinking:

- I wonder why . . .
- I still have a question about . . .
- So you are saying that . . .

For example, for Problem 2.2 in the grade 7 unit *Completely Rational*, using the sentence stem "I believe my answer is correct because . . ." can help students make sense of their thinking around subtracting with integers.

Stop and Jot

Stop and Jot is a teacher move that can be used at any phase of a CMP lesson to allow students the opportunity to pause and process what they are learning before moving on in a lesson or before sharing their thoughts with their peers (The Teacher Toolkit, n.d.). Additionally, this strategy can be used as a formative assessment to gauge students' level of comprehension and guide teachers' next steps in a lesson or series of lessons. A teacher initiates a Stop and Jot by pausing the lesson at a critical moment and posing a question to students. Students then independently respond in writing to the prompt in a designated writing space (e.g., a sticky note, index card, capture sheet, special notebook).

Below are some examples of moments in a CMP lesson that Stop and Jot could be used:

- before launching a new investigation or problem to activate prior knowledge
- during the Launch of a problem to have students make predictions
- at the beginning of the Explore phase before working with a partner or group
- during the middle of a Summarize discussion to provide an opportunity for students to make sense of others' strategies or to make connections between several different strategies
- at the end of the problem to provide closure, check understanding, and clarify any misunderstandings
- during the Launch, Explore, or Summarize to pose a question to students you want them to consider

For example, in Problem 1.2 in the grade 8 unit *Thinking With Mathematical Models*, having students Stop and Jot about their thoughts about the relationships in the data from the experiment before talking with others will foster a richer summary as a class.

Think, Pair, Share

The *Think, Pair, Share* strategy can be used in any phase of a CMP lesson. The strategy is intended to encourage and support equitable conversations where students are given time to process and engage with ideas independently, with a partner, and in a whole-group discussion (EL Education, 2014). The teacher initiates this strategy by giving students text, a mathematics problem, a question, a piece of student work, etc. to examine and process independently. After

the independent examination, students are placed in pairs for partner discussion. Each partner takes turns sharing their thoughts, ideas, and questions. Finally, the teacher facilitates a whole-class discussion where students share the ideas they came up with or heard from their partners.

Teachers might choose to modify this and refer to the strategy as Think, Pair, Share. The modification occurs in the first step, where the students not only "think" about the text, problem, question, or piece of student work but also record their thoughts and work to bring to their partner discussions.

Below are some examples of moments in a CMP lesson that Think, Pair, Share could be used:

- during the Launch of a problem to activate prior knowledge about the context or the content of the problem/investigation
- during the Launch phase when reading introductory text to a problem
- as a way to structure the entire Explore phase of a lesson
- during the middle of a Summarize discussion to provide an opportunity for students to make sense of others' strategies or to make connections between several different strategies
- at the end of the problem to provide closure, check understanding, and clarify any misunderstandings

For example, in Problem 2.1 of the grade 6 unit *Number Connections*, in the Launch phase, having students think about using how the size of the Ferris wheel might affect their ride before they work with their partner will help each person to have something to bring to the conversation.

Three Reads

Three Reads is a strategy CMP teachers can use in the Launch phase of a lesson. This strategy is intended to engage students in making sense of the language in high-cognitive-demand tasks prior to actively exploring the task (SFUSD Mathematics Department, 2015). Three Reads focuses on the importance of understanding a task's context and mathematical content before moving to find a solution. When using this strategy, teachers present students with the task, leaving out the problem question. Students are then asked to read the task three times to focus their attention on the context and mathematical content of the task.

During the first read, students independently read the task to become familiar with the context. The teacher prompts students to answer the question "What is this task about?" and ask clarifying questions.

Students are asked to read the task a second time to become familiar with the mathematical content of the task. The teacher prompts students to answer the question "What are the given quantities in this task?" Note, to answer this question, the teacher will need to facilitate a class discussion about the meaning of the word *quantities*.

For the final read, students read the task and are asked to create mathematical questions using the information given in the task. The teacher then facilitates a class discussion focused on the questions posed by students and the potential to answer the questions with the provided information.

After completing the three reads, the teacher shares the actual question posed in CMP, and the students move into the Explore phase of the lesson. Often, the question posed in the curriculum has already been posed by a student during the third read. The teacher may use this opportunity to highlight the student who posed the question that matches the question in CMP.

For example, when launching Problem 1.3 of the grade 7 unit *Moving Straight Ahead*, a teacher might present the following task to students without the problems from the Initial Challenge to implement the Three Reads strategy.

The students in Ms. Chang's class are trying to estimate how much money they might raise for charity by doing a walkathon. Each participant in the walkathon must find sponsors to pledge a certain amount of money for each kilometer they walk. The class refers to these as pledge plans. Each student found sponsors who pledge to donate money according to the following descriptions.

From Grade 7, Problem 1.3 in the *Moving Straight Ahead* Unit

Name	Pledge Plan
Leanne	$10 regardless of how far she walks
Gilberto	$2 per kilometer (km)
Alana	$5 donation plus 50¢ per kilometer

Turn and Talk

Turn and Talk is a strategy that can be used during the Launch and Summarize phases of a CMP lesson. The teacher initiates this strategy by asking students to turn to a partner to discuss a question, idea, or argument (West, 2016). Turn and Talk can be used for many purposes, including but not limited to

- reflecting on an idea or strategy presented by a student or group;
- activating prior knowledge;
- rehearsing a response prior to sharing with the whole class;
- recalling information from a discussion;
- explaining or restating a classmate's strategy;
- generating questions; and
- expanding on an idea being discussed.

During a Turn and Talk, the teacher must ensure that all students have a partner to talk with (West, 2016). The teacher must hold students accountable for either talking or listening for the duration of the Turn and Talk. While students are talking, the teacher should listen in on conversations without interrupting or interfering. This allows the teacher to hear students' thinking and misconceptions that they might want to highlight during a whole-class discussion.

Turn and Talk increases active participation and promotes sharing ideas openly. Additionally, this strategy provides reticent students opportunities to rehearse responses before sharing them with the whole class.

For example, for Problem 4.3 in the grade 8 unit *Function Junction*, having students Turn and Talk to discuss the functions they have studied prior to doing the *Functions Five* game in the Initial Challenge can activate prior knowledge to bring to the game.

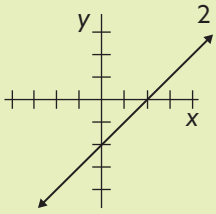
From Grade 8, Problem 4.3 in the *Function Junction* Unit

Functions Five Game Board

1

$y = (x + 2)^2 - 1$

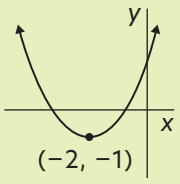
2



3

x	0	1	2	2
y	1	2	4	8

4



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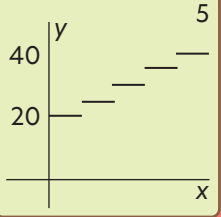


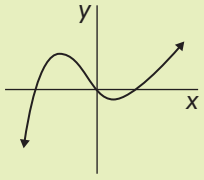
6

$y = x^2 + 1$

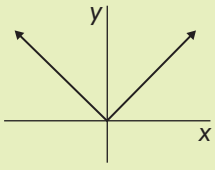
7

$y = (x + 1)(x - 3)$
$(x + 2)$

8



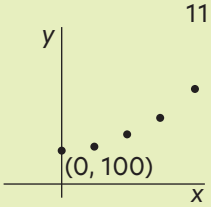
9



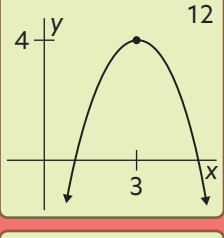
10

x	1	2	3	4
y	5	8	11	14

11



12



13

x	0	1	2	3
y	−2	0	2	4

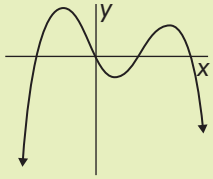
14

$y = x^2(x + 1)(x - 3)$

15

$y = -(x - 1)^2$

16



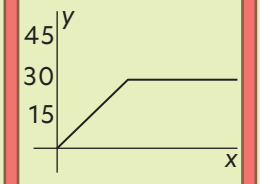
17

x	1	2	3	4
y	2	7	12	17

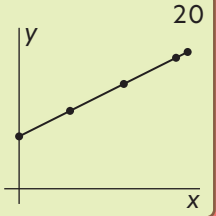
18

$y = x(x + 2)$

19



20



21

$y = 100(1.05)^x$

22

$f(x) = 2x - 3$

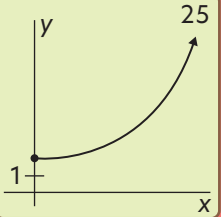
23

$f(x) = \frac{1}{2}x + 2$

24

$f(x) = 3(x + 2)$

25



Wait Time

Wait Time is a strategy teachers can use during any phase of a CMP lesson that gives all students time to process a question or comment made by another student. After asking a question or asking students to respond to a peer's reasoning, the teacher allows at least 10 seconds for students to think before calling on someone to respond (Saphier & Haley-Speca, 2015).
If no one offers an answer or response after the 10-second wait time, the teacher should extend the wait time rather than scaffold the question or tell the students the answer they were looking for. Wait Time can also be used when the teacher calls on a student to give an answer. Giving Wait Time in this instance allows students to organize their thoughts before orally sharing them with the class (Chapin et al., 2013).

For example, for Problem 3.1 in the grade 6 unit *Covering and Surrounding*, answering the questions "What kind of constraints make drawing a triangle easy?" and "What kind of constraints make drawing a triangle difficult?" can require some processing time. Students will need time to synthesize all they've encountered in the problem. Give students time without offering suggestions.

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GUIDE AID
4.B AILN Detailed Framework

PROBLEM-SOLVING ENVIRONMENT	
Fostering Mathematical Sensemaking, Connections, and Applications	*Opportunities for students to solve problems and **make sense of the embedded mathematics and to connect and apply these understandings** to develop deeper mathematical understandings and ways of thinking* Characteristics: • allows multiple entry points • provides supports that assists them in the problem. • includes ways to connect experiences and thinking to the problem • provides supports to communicate reasoning with others • fosters physical and social space to explore and solve problems • allows welcoming, affirming feeling while having high expectations • provides resources and time for mathematical ideas and concepts

AGENCY, IDENTITY, AND OWNERSHIP	
Supporting Knowers, Doers, and Creators of Mathematics	*Opportunities for students to **participate as knowers, doers, and creators of mathematics** to develop deep mathematical understandings* Characteristics: • showcases student work and voices to highlight mathematical thinking • encourages reflection on individual and collective thought processes • utilizes student work from prior classes as examples of mathematical thinking • promotes personal connections to mathematical contexts • integrates current events, cultural elements, and local figures into discussions • actively invites student contributions during class discussions

LANGUAGE	
Communicating Mathematical Ideas	*Opportunities for students to **communicate mathematical ideas** to develop deep mathematical understandings* Characteristics: • provides multiple opportunities for students to express and share mathematical thinking • acknowledges the social nature of math learning and problem solving • encourages students to articulate their mathematical thoughts • utilizes diverse forms of communication to deepen understanding • enhances mathematical fluency through active engagement in problem solving and discussion

PORTRAYAL	
Accessing and Engaging in Mathematics	*Opportunities for students to **engage, represent, and interpret with mathematics in a variety of ways** to develop deep mathematical understandings* Characteristics: • offers multiple access points to problems to enhance student connection and engagement with mathematical ideas • allows students to develop and represent their understanding in various formats • utilizes images as anchors for embedding and retrieving understanding of key mathematical concepts • presents diverse examples to support the portrayal of mathematics

TIME	
Developing Mathematical Ways of Thinking	*Opportunities for students to **develop mathematical ways of thinking** to develop deep mathematical understandings* Characteristics: • allows student exploration, sharing, and summarization in problem solving • embeds design elements for understanding mathematical concepts • provides a sequence of problems centered on key mathematical concepts • aids in developing conceptual and procedural knowledge, moving from informal to formal understanding over problems, units, and grades

GUIDE AID
3.H CMP Instructional Model Overview

	Teacher's Role	Students' Role
Launch	• Introduce new concept if needed • Review old concept if needed • Help students understand the context • Introduce the challenge	• Notice and wonder what they know and what they need to do • Make prediction or conjecture • Ask clarifying questions
Explore	• Observe individual understandings • Listen to student reasoning • Ask questions to encourage, connect, redirect, and extend • Identify student understanding and strategies to use in the Summarize	• Look for patterns • Make conjectures • Share ideas • Develop strategies • Create arguments to support their reasoning and solution • Ask questions to themselves, classmates, and teacher
Summarize	• Observe differences and similarities • Critique reasoning • Discuss strategies; refine strategies • Collect, organize, document reasoning • Connect ideas to prior work • Ask new questions	

GUIDE AID 3.1

Questions Students Can Ask in the Explore

These are possible questions that students can ask each other as their group is working. Not all of the questions are necessary each time.

Moving Forward	Recording Ideas	Staying on Time	Getting Ready to Share
• What do we have to do? OR Does anyone have questions about what we need to do? • Does anyone have a conjecture or prediction for what will happen? • Do you want to think about it by yourself for a minute before we talk as a group? • Anyone have ideas where to start? • Can we draw a picture or diagram? Or write an expression? • Do we see any patterns? • Can we explain or justify our thinking? • Did everyone get a chance to share their thinking about the problem? • Have we answered the question(s)?	• What do we want to put in the notes? • Here is what I have. Does anyone want to add more? • Does this (looking at notes) represent our group's work? • Do you think we can work with these notes to explain our group's ideas?	• How much total time do we have to work? Would you like me to remind the group that we're near the end? With 5 minutes left? With 2 minutes left? Some other amount of time? • Do you think that we are done? • Did we answer all of the questions? • Do we need more time?	• What is the most important thing for us to say when we report out to the whole class? • What other things should we say? • Do we have any questions to ask the other groups (or our teacher)?