UNIT ALIGNMENTS

GOALS, ARC OF LEARNING™, STANDARDS, NOW WHAT DO YOU KNOW?, AND EMERGING MATHEMATICAL IDEAS

Investigation 1: Organizing a Bike Tour: Variables, Tables, and Graphs

Goals

Relationships Between Variables. Begin informal understanding of variables and how they are related.

- Explore problem situations that involve variables and relationships including situations that change over time
- Recognize variables in a real-world situation, identify the dependent and independent variables, and describe how they are related in a situation
- Make sense of the "stories" told by patterns in tables and coordinate graphs of numeric (x, y) data and equations and make connections across the representations
- Represent relationships between two variables with one operation ($y = ax$ and $y = b + x$) using words/stories, data tables, graphs, and equations
- Describe advantages and disadvantages of using words, tables, graphs, and equations to represent a relationship between two variables and to answer questions about the relationship
- Solve problems that involve variables to answer questions about one variable given the value of the associated variable when represented using tables, graphs, equations, or words (a story context)

Expressions and Equations. Begin informal understanding of expressions and equations.

- Recognize that equations describe a relationship between two variables
- Represent real-world relationships, stories, involving two variables and one operation ($y = ax$ and $y = b + x$) with an equation and describe in words a relationship given in the form of an equation such as $y = ax$ and $y = b + x$
- Recognize that expressions like ax or $b + x$ represent a relationship of a quantity or mathematical pattern
- Identify parts of an equation using mathematical terms (*sum*, *term*, *product*, *factor*, *quotient*, *coefficient*)
- Use an equation such as $y = ax$ and $y = b + x$ to determine the value of one variable given the value of the other using numeric guess and check, tables of (x, y) values, and graphs

Arc of Learning™, Standards, Now What Do You Know?, and Emerging Mathematical Ideas

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<table>
<tr><td colspan="2">Problem 1.1. Organizing a Bike Tour Experiment: Variables and Tables
Arc of Learning™: Introduction
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How does a table help you make sense of the relationship between two variables in a situation?</td><td>Emerging Mathematical Ideas
Begin making sense of a problem situation that involves a relationship between two variables represented in a table and connecting it to a description of the relationship in words and vice versa.
Notice in this context:
• As time increases, the number of jumping jacks increases.
• The change in the number of jumping jacks is the same or close to the same for each interval of 10 seconds, and words like consistent and constant can be used to describe relationships like this one.
• In the table, time values are recorded in consistent/constant intervals, and the jumping jacks are also growing by a rather consistent amount.
• Tables help you to quickly notice differences or changes in the number of jumping jacks for each 10 seconds (rates).</td></tr>
<tr><td colspan="2">Problem 1.2. Organizing a Bike Tour: Variables, Tables, and Graphs
Arc of Learning™: Introduction, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
Describe how the pattern of change relationship between the two variables—time and number of jumping jacks—is represented in a table and in a graph.</td><td>Emerging Mathematical Ideas
Continue working to understand how the relationship between two variables can be represented in a table, and introduce graphs as another way to represent the relationship between two variables.
Notice in this context:
• As values for the top row/variable increase (time), the values in the second row also increase (number of jumping jacks).
• The change in the number of jumping jacks (second row) is the same for each interval of 10 seconds.
• Tables let you quickly compare how both variables are changing.
Explore how a relationship between two variables can be represented on a graph.
Notice in this context:
• The points are moving up as the time increases.
• Graphs, with the same scales, may have points moving up faster than on other graphs depending on the relationship between the variables.
• Two graphs can represent the same relationship but look different. The scale for the axes makes a difference in the way the graph looks, in particular how fast the points look like they are moving up.</td></tr>
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Problem 1.3. Atlantic City to Lewes to Chincoteague: Time, Rate, and Distance

Arc of Learning™: Introduction, Exploration

Standards: See your state alignment chart.

Now What Do You Know?	**Emerging Mathematical Ideas**
Describe how distance changes over time. How is this pattern of change shown in tables and graphs?	Extend thinking about the relationships between variables to patterns of change that are not constant. Explore the biking context, with more nuanced relationships, working to • identify variables in a real-world situation; • make connections between coordinate points on a graph and values from a table; • interpret data points, thinking about how values in the table and on the graph connect to narratives about the data; • describe the relationship between the variables as displayed in a table and a graph; and • analyze why the changes in time can be constant but the changes in distance are not.

Problem 1.4. Chincoteague Island to Norfolk: Stories, Tables, and Graphs

Arc of Learning™: Introduction, Exploration

Standards: See your state alignment chart.

Now What Do You Know?	**Emerging Mathematical Ideas**
Describe how the relationship between two variables is shown in written notes, tables, and graphs. What are the advantages and disadvantages of each representation?	Extend thinking about the relationships between variables when given limited notes about the context. Begin working to • identify variables in a real-world situation; • construct a graph and a table given a written description of a relationship between two variables; • compare the use of a table and written notes to describe the relationship between time and distance; • informally use rates to construct a table given the narrative; and • make sense of instances when there is a change in the time but no corresponding change in the distance and how this looks in a table and graph.

Mathematical Reflection

Arc of Learning™: Exploration

Standards: See your state alignment chart.

Mathematical Reflection	**Emerging Mathematical Ideas**
What are the advantages and disadvantages of using different representations to show the relationship between two variables?	Develop an understanding for key aspects of representing and making sense of variable relationships given in tables, graphs, and words. Notice: • Tables give explicit numerical values for the variables, and this information from graphs requires reading scales and sometimes imprecise value estimates. • Graphs show an overall trend in the data and make it easy to identify changes in a pattern. • Narratives tell what is happening yet can be vague or incomplete, requiring one to make interpretations.

Investigation 2: Determining Tour Needs: Analyzing Relationships Among Variables

Goals

Relationships Between Variables. Begin informal understanding of variables and how they are related.

- Explore problem situations that involve variables and relationships including situations that change over time
- Recognize variables in a real-world situation, identify the dependent and independent variables, and describe how they are related in a situation
- Make sense of the "stories" told by patterns in tables and coordinate graphs of numeric (x, y) data and equations and make connections across the representations
- Represent relationships between two variables with one operation ($y = ax$ and $y = b + x$) using words/stories, data tables, graphs, and equations
- Describe advantages and disadvantages of using words, tables, graphs, and equations to represent a relationship between two variables and to answer questions about the relationship
- Solve problems that involve variables to answer questions about one variable given the value of the associated variable when represented using tables, graphs, equations, or words (a story context)

Expressions and Equations. Begin informal understanding of expressions and equations.

- Recognize that equations describe a relationship between two variables
- Represent real-world relationships, stories, involving two variables and one operation ($y = ax$ and $y = b + x$) with an equation and describe in words a relationship given in the form of an equation such as $y = ax$ and $y = b + x$
- Recognize that expressions like ax or $b + x$ represent a relationship of a quantity or mathematical pattern
- Identify parts of an equation using mathematical terms (*sum*, *term*, *product*, *factor*, *quotient*, *coefficient*)
- Use an equation such as $y = ax$ and $y = b + x$ to determine the value of one variable given the value of the other using numeric guess and check, tables of (x, y) values, and graphs

Arc of Learning™, Standards, Now What Do You Know?, and Emerging Mathematical Ideas

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<table>
<tr><td colspan="2">Problem 2.1. Renting Bicycles: Independent and Dependent Variables
Arc of Learning™: Introduction, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
We can examine the relationship between the dependent and independent variables. What information was easier to get from the table? From the graph? Explain why.</td><td>Emerging Mathematical Ideas
Use informal understandings of rate and proportional reasoning to explore the relationship between an independent variable (number of bicycles) and a dependent variable (total cost).
Use the language of independent and dependent variables.
Explore ways to determine
• which representation of the relationship is most useful based on the information needed and the type of relationship; and
• values not specifically given on a table or graph using rates and proportional reasoning informally.</td></tr>
<tr><td colspan="2">Problem 2.2. Finding Customers: More Variables
Arc of Learning™: Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
How is the pattern of change between the dependent and independent variables in this problem the same as those in Investigation 1 and Problem 2.1? How is it different?</td><td>Emerging Mathematical Ideas
Explore reasoning with relationships between two variables given in tables and graphs with the added complexity of a context that is a decreasing pattern of change.
Explore ways to
• use the language of independent and dependent variables;
• use data for a relationship given in table form to create a corresponding graph with appropriate scales;
• reason about a decreasing relationship represented in a table and in a graph; and
• compare the decreasing pattern of change in this problem with the increasing patterns used earlier, noticing similarities and differences in their graphs and tables.</td></tr>
<tr><td colspan="2">Problem 2.3. What's the Story?: Interpreting Graphs
Arc of Learning™: Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
What do you know about the relationship between the dependent and independent variable from a rising graph? A level graph? A falling graph?</td><td>Emerging Mathematical Ideas
Establish key aspects of relationships that are increasing, decreasing, and/or not changing when represented in words and graphs.
Explore ways to
• identify dependent and independent variables based on the context;
• use the idea of proportion to make sense of graphs;
• understand that a "rising" graph means that as the independent variable is increasing, the dependent variable is increasing;</td></tr>
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	• understand that a level graph means the dependent variable is not changing even as the independent one is; and • understand that a "falling" graph means the dependent variable is decreasing as the independent variable is increasing.
Mathematical Reflection **Arc of Learning™: Exploration** **Standards:** See your state alignment chart.	
Mathematical Reflection What are the advantages and disadvantages of using different representations to show the relationship between two variables?	**Emerging Mathematical Ideas** Compare representations looking for generalizations. Explore ways to determine how • tables give explicit numerical values for the variables; • getting specific values from graphs requires reading scales and is sometimes only an estimate; • graphs give an overall picture of the relationship, including places where the pattern of change shifts; and • narratives explain what is happening with key ideas and tell a story but sometimes can be incomplete.

Investigation 3: Returning Home: Relating Variables, Expressions, and Equations

Goals

Relationships Between Variables. Begin informal understanding of variables and how they are related.

- Explore problem situations that involve variables and relationships including situations that change over time
- Recognize variables in a real-world situation, identify the dependent and independent variables, and describe how they are related in a situation
- Make sense of the "stories" told by patterns in tables and coordinate graphs of numeric (x, y) data and equations and make connections across the representations
- Represent relationships between two variables with one operation ($y = ax$ and $y = b + x$) using words/stories, data tables, graphs, and equations
- Describe advantages and disadvantages of using words, tables, graphs, and equations to represent a relationship between two variables and to answer questions about the relationship
- Solve problems that involve variables to answer questions about one variable given the value of the associated variable when represented using tables, graphs, equations, or words (a story context)

Expressions and Equations. Begin informal understanding of expressions and equations.

- Recognize that equations describe a relationship between two variables
- Represent real-world relationships, stories, involving two variables and one operation ($y = ax$ and $y = b + x$) with an equation and describe in words a relationship given in the form of an equation such as $y = ax$ and $y = b + x$

- Recognize that expressions like ax or $b + x$ represent a relationship of a quantity or mathematical pattern
- Identify parts of an equation using mathematical terms (*sum*, *term*, *product*, *factor*, *quotient*, *coefficient*)
- Use an equation such as $y = ax$ and $y = b + x$ to determine the value of one variable given the value of the other using numeric guess and check, tables of (x, y) values, and graphs

Arc of Learning™, Standards, Now What Do You Know?, and Emerging Mathematical Ideas

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<table>
<tr><td colspan="2">Problem 3.1. Returning Home: Equations with One Operation
Arc of Learning™: Introduction, Exploration, Introduction
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
Describe how an equation might be useful to answer questions about the situation. What are the advantages of using a table, graph, or equation to make a prediction and solve a problem?</td><td>Emerging Mathematical Ideas
Exploration of the relationships between two variables continues with the introduction of equations to represent relationships from contexts of the form $y = ax$.
Explore ways to
• use a context to explain how to write an equation of the form $y = ax$ (a proportional relationship), by relating the pattern in a table to a general description in words then to an equation using numbers (rates) and variables;
• begin to connect the coefficient of the independent variable with the rate of change in the dependent variable;
• begin to meaningfully use terms such as equation, expression, coefficient, and products; and
• informally evaluate equations for specific values of their variables.</td></tr>
<tr><td colspan="2">Problem 3.2. Planning the Next Tour: More Equations with One Operation
Arc of Learning™: Exploration, Exploration
Standards: See your state alignment chart.</td></tr>
<tr><td>Now What Do You Know?
Compare the equations in this problem to those in Problem 3.1. How are they similar? How are they different? Describe how an equation might be useful to answer questions about a situation.</td><td>Emerging Mathematical Ideas
Explore contexts where the relationship between two variables can be described with equations of the form $y = x + b$ or $y = x - b$.
Explore ways to
• use the context to explain how to write an equation of the form $y = x$ + b or $y = x - b$ (sums or differences) by relating the pattern described in words to the equation using numbers and variables;
• informally evaluate equations at specific values of their variables using the context to make sense of the question and answer;
• notice that contexts and equations in Problems 3.1 and 3.2 involve one operation;</td></tr>
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	• distinguish between contexts that involve addition or subtraction of a constant amount and those that involve multiplication by a constant amount; and • begin to meaningfully use terms such as *sum* and *difference* when referring to equations/expressions.
Problem 3.3. Planning Ahead: Connecting Equations with Tables and Graphs **Arc of Learning™: Exploration, Exploration** **Standards:** See your state alignment chart.	
Now What Do You Know? How do you know if a table, a graph, and an equation represent the same situation? How can you use a table, a graph, an equation to answer specific questions for a situation?	**Emerging Mathematical Ideas** Continue to explore contexts where the relationship between two variables can be described with equations using one or two operations and with tables and graphs. Explore ways to • begin to distinguish between contexts that involve addition or subtraction of a constant amount, those that involve multiplication by a constant amount, and those that involve both addition/subtraction and multiplication ($y = b + ax$ or $y = b - ax$); • use the context to explain how to write an equation of the form $y = b + ax$ or $y = b - ax$ by relating the pattern described in words to the equation using numbers and variables; • informally evaluate equations at specific values of their variables using the context to make sense of the question and answer; • explain how to use tables and graphs to answer questions for specific values of the variables; and • make connections among different representations (contexts, tables, graphs, and equations) for a relationship.
Mathematical Reflection **Arc of Learning™: Exploration, Exploration** **Standards:** See your state alignment chart.	
Mathematical Reflection What are the advantages and disadvantages of using different representations to show the relationship between two variables?	**Emerging Mathematical Ideas** Begin to articulate that • tables can give explicit numerical values for the variables; • getting specific values for the variables from graphs requires reading scales and is sometimes only an estimate; • graphs give an overall picture of the relationship, including places where the pattern of change shifts; • narratives explain what is happening with key ideas and tell a story but sometimes can be incomplete; and • equations are an efficient way to describe the relationship between two variables and can be used to find exact values for the variables.